

Online Appendix to
Large Multiproduct Exporters Across Rich and Poor
Countries: Theory and Evidence

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1 Theoretical Appendix

1.1 Two Asymmetric Countries

This section describes how the relevant variables considered in the paper vary across rich and poor countries. Proving analytically the main results of the papers can be done in the neighborhood of the symmetric country equilibrium. The details are available upon request. The reason why analytical comparative static exercise are difficult lies with our definition of cost cutoff. In most model of international trade the cost cutoff to reach a destination can be expressed as a function of the domestic cost cutoff. Such a result cannot be achieved in a model with cannibalization effects, because the cost cutoff of reaching a destination depends on the firms' market shares. Hence, I rely on numerical methods to solve the model.

It is useful to rewrite the equations that characterize the equilibrium of the model:

$$y_h c_h s_{hh}^\theta (1 - s_{fh}) = \tau c_f y_f s_{fh}^\theta (1 - s_{hh}) \quad (1)$$

$$y_f c_f s_{ff}^\theta (1 - s_{hf}) = \tau c_h y_h s_{hf}^\theta (1 - s_{ff}) \quad (2)$$

$$(s_{hh}^2 + \theta s_{hh}) y_h L_h + (s_{hf}^2 + \theta s_{hf}) y_f L_f = F(\theta + 1) y_h \quad (3)$$

$$(s_{ff}^2 + \theta s_{ff}) y_f L_f + (s_{fh}^2 + \theta s_{fh}) y_h L_h = F(\theta + 1) y_f \quad (4)$$

$$M_h s_{hh} + M_f s_{fh} = 1 \quad (5)$$

$$M_h s_{hf} + M_f s_{ff} = 1 \quad (6)$$

$$M_j s_{ji} y_i L_i = M_i s_{ij} y_j L_j \quad (7)$$

1.1.1 Product Scope and Cannibalization Effects in Partial Equilibrium

To study how scope and market share of superstars vary with the per capita income of the destination, keeping size and productivity constant, I conduct the following exercise. I let $L_f/L_h = c_f/c_h = 1$ and vary $y_f \in [1, 2]$. I solve the system of equations given by (1), (2), (3) and (4) (the relative cutoffs across destinations and the zero profit conditions) and record the relevant variables. The other parameters of the model are: $\theta = 1$, $\tau = 2$, $F = 0.1$, $\bar{q} = 0.001$.

Table 1 shows the results from the simulations. Both the scope and the market share of home firms in the foreign economy is increasing in the per capita income y_f .

1.1.2 Product Scope and Cannibalization Effects in General Equilibrium

To study in general equilibrium how scope, prices, and cannibalization effects vary with the per capita income of the destination, I consider two exercises:

Table 1: Varying per capita income

	δ_{hf}	s_{hf}
y_f	6.176*** (0.070)	0.025*** (0.001)
R^2	0.99	0.89
# Observations	100	100

I solve the model for $y_f \in [1, 2]$ and collect the relevant variables. The table shows the coefficients estimated from the univariate regression of a variable (in the column) against y_f and a constant.

1. Home and Foreign economy have the same size L but their relative efficiency c_f/c_h is different than 1.
2. Home and Foreign economy have the same efficiency c but their relative sizes varies, keeping the aggregate world size constant¹.

Consider the first scenario. I let $c_f/c_h \in [0.25, 2]$ with incremental steps of 0.01. I solve the system of equations given by (1), (2), (3), (4), (5), (6), and (7). Similarly, in the second scenario I let L_f/L_h vary between $[0.3, 1.7]$ letting $L_h + L_f = 2$, with incremental steps of 0.01. The other parameters of the model are: $\theta = 1$ in the benchmark case², $\tau = 2$, $F = 0.1$, $\bar{q} = 0.001$. In the first scenario, I set $L = 1$, while in the second $c = 1$. The results are described in the following subsections.

Table 2 shows how the relevant variables vary with $\frac{c_f}{c_h}$. Results are summarized in table (2). First, the more productive the foreign economy is compared to the home economy (the lower c_f/c_h), the higher the foreign per capita income. In this scenario, the more productive economy is also the richer economy.

To fix ideas, suppose the foreign economy gets richer. Home firms export more varieties in the richer economy. In fact, even though they lose market shares (s_{hf} falls), because of the stronger competition from the more productive foreign firms, the positive effect of per capita income on δ_{hf} dominates. A similar result holds for the relative price of a good that is exported from home to the foreign economy, relative to the price of the same good in the home economy. Foreign firms gain more market share as their productivity increases, and their domestic scope expands.

Let us now consider the second scenario, in which the two economies have the same productivity but their relative size changes (Results in Table 3). The larger the foreign economy, the higher its per capita income. In fact, as more foreign firms enter, and each of them focuses on their core competence, workers' productivity increases. Home firms export more varieties in the larger economy and this result is driven by the increase in per capita income. In fact, when we control

¹Increasing the size of a country keeping the other one constant generates different results explored in previous versions of this paper. Details available upon request.

²Results are qualitatively similar for different values of $\theta = 0.8, 3, 6, 9$

Table 2: Varying relative productivities

	y_f/y_h	δ_{hf}	s_{hf}	δ_{ff}	s_{ff}	p_{hf}/p_{hh}
c_f/c_h	-0.994*** (0.041)	-0.825*** (0.004)	0.038*** (0.000)	-12.623*** (0.384)	-0.034*** (0.000)	-0.029*** (0.000)
R^2	0.77	1.00	0.97	0.86	0.98	0.99
# Observations	176	176	176	176	176	176

I solve the model for $c_f/c_h \in [0.25, 2]$ and collect the relevant variables. The table shows the coefficients estimated from the univariate regression of a variable (in the column) against c_f/c_h and a constant. Example: the first entry is the coefficient from regressing y_f/y_h on c_f/c_h and a constant.

for y_f/y_h , the size of the destination has a negative effect on the scope of home exporters. The reason for that is that the market share of home firms in the foreign economy falls.

Table 3: Varying relative sizes

	y_f/y_h	δ_{hf}	δ_{hf}	s_{hf}	δ_{ff}	s_{ff}	p_{hf}/p_{hh}
L_f/L_h	0.416*** (0.008)	0.468*** (0.029)	-1.022*** (0.008)	-0.021*** (0.000)	-2.359*** (0.019)	-0.037*** (0.000)	-0.125*** (0.001)
y_f/y_h			3.586*** (0.018)				
R^2	0.98	0.80	1.00	0.99	1.00	0.99	0.99
# Observations	71	71	71	71	71	71	71

I solve the model for $L_f/L_h \in [0.3, 1.7]$ and collect the relevant variables. The table shows the coefficients estimated from the univariate regression of a variable (in the column) against c_f/c_h and a constant. Example: the first entry is the coefficient from regressing y_f/y_h on c_f/c_h and a constant. Standard Errors in parenthesis.

Finally, let us consider how cannibalization effects vary across the two countries. As shown in the paper, cannibalization effects faced by domestic firms are related to the market share of the typical firm s_{jj} . To study how cannibalization effects vary across countries, let us focus on the ratio between the domestic market shares of firms in the two countries s_{ff}/s_{hh} . If the ratio increases, domestic firms in the foreign economy face stronger cannibalization effects than domestic firms in the home economy. Table 4 illustrates the results of the simulations. The more productive an economy is, the stronger the cannibalization effects faced by its firms. Suppose foreign firms are the most productive. As foreign firms compete with less productive home firms, s_{ff} is larger than s_{hh} . Moreover, the larger an economy, the smaller the cannibalization effects faced by its firms. A large economy is characterized by a larger number of firms: as these firms are identical, their market share is smaller than the market share of firms from a smaller economy.

1.1.3 Welfare Gains Across Rich and Poor Countries

How do welfare gains compare across rich and poor countries? To answer this question, we need to study two components of our welfare formula. In particular, how do 1) cannibalization effects (s_{jj}), and 2) the change in s_{jj} vary across rich and poor countries? Table 5 summarizes the result,

Table 4: Varying relative size

	s_{ff}/s_{hh}	s_{ff}/s_{hh}
c_f/c_h	-0.384***	
	(0.006)	
L_f/L_h		-0.725***
		(0.004)
R^2	0.96	1.00
# Observations	176	71

Coefficients on the univariate regression. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

showing how changes in the fundamental variables of the model (efficiency and size) affect income differences, cannibalization effects and welfare gains from trade.

Table 5: Welfare Gains from Trade

	$\frac{y_h}{y_f}$	$\frac{s_{hh}}{s_{ff}}$	$\frac{-d \ln s_{hh}}{-d \ln s_{ff}}$	$\frac{d \ln W_h}{d \ln W_f}$
$\frac{c_h}{c_f}$	-	-	+	+
$\frac{L_h}{L_f}$	+	-	-	-

Effect of increasing relative efficiency $\frac{c_h}{c_f}$ and size $\frac{L_h}{L_f}$ on $\frac{y_h}{y_f}$, $\frac{s_{hh}}{s_{ff}}$, $\frac{-d \ln s_{hh}}{-d \ln s_{ff}}$ and $\frac{d \ln W_h}{d \ln W_f}$.

Let us start with the first row of table 5. Suppose that the home and foreign economy have the same size L but they differ in terms of productivity. In particular consider a case in which the foreign economy is more productive than the home one ($\frac{c_h}{c_f} > 1$). As previously described, consumers in the foreign economy enjoy a higher per capita income than home consumers. In addition, because foreign firms are more productive than home firms, $s_{ff} > s_{hh}$. Hence, cannibalization effects faced by domestic firms are stronger in the more efficient and richer foreign economy.

The model predicts that given the same change in s_{jj} the foreign economy would gain more than the home economy. However, given a small reduction in the trade costs τ , $d \ln s_{hh}$ is larger, in absolute terms, than $d \ln s_{ff}$. A reduction in trade cost makes the exporters to a destination relatively more efficient than before. This shock erodes the market shares of domestic firms: the less productive the domestic firms, the larger the loss of market share. To summarize, firms in the more productive economy face stronger cannibalization effects, but their market share falls by less after a reduction in τ than the domestic market share of firms in the less productive economy. The second effect dominates: the less productive economy gains more from trade.

Let us now consider a scenario in which home and foreign economy have the same efficiency level c , but their sizes differ. In particular, suppose that $L_h > L_f$. As seen in the previous section, the larger economy also has the higher per capita income. In this case, firms in the smaller economy face the stronger cannibalization effects and experience the largest decrease in s_{jj} . Hence, welfare gains from trade are larger for small economies.

To derive the results I use the following algorithm. For each $c_f/c_h \in [0.25, 2]$ $L_f/L_h \in [0.3, 1.7]$ I consider an equilibrium with $\tau = 2$ and one with $\tau = 1.98$ and study the change in the domestic market share of the typical domestic firm as well as the welfare gains from trade.

Table 6: Welfare gains

	$d \ln s_{ff}/d \ln s_{hh}$	$d \ln W_f/d \ln W_h$	$d \ln s_{ff}/d \ln s_{hh}$	$d \ln W_f/d \ln W_h$
$d \ln c_f/d \ln c_h$	0.990*** (0.003)	0.936*** (0.003)		
$d \ln L_f/d \ln L_h$			-5.184*** (0.386)	-5.467*** (0.415)
R^2	1.00	1.00	0.72	0.72
# Observations	176	176	71	71

Coefficients on the univariate regression. Std. error in parenthesis. ***: significant at 99%, ** at 95%, * at 90%.

Table (6) shows the results. The less productive economy and the smaller economy experience the largest change in the domestic market share of the typical firm. Such a change offsets the smaller cannibalization effects that the firms in this country are facing, yielding larger welfare gains. Overall, poorer economies (either because of the lower productivity or the smaller size) gain more from trade than richer economies.

1.2 Stylized Facts from Literature

In this section, I derive further predictions from the model that are consistent with established regularities documented in the literature.

Mayer, Melitz and Ottaviano (2014)

The model is consistent with the firm-level prediction documented by Mayer et al. (2014): the ratio of sales of the core product to the second best product, increases with competition. Using the revenues from a variety ω $r_{kij}(\omega) = x_{kij}(\omega)p_{kij}(\omega)$, for any $\omega < \omega'$:

$$\frac{r_{kij}(\omega)}{r_{kij}(\omega')} = \frac{\delta_{kij}^{\frac{\theta}{2}} - \omega^{\frac{\theta}{2}}}{\delta_{kij}^{\frac{\theta}{2}} - \omega'^{\frac{\theta}{2}}} \quad (8)$$

which is decreasing in δ_{kij} . For a market share less than 50%, higher competition, represented by a smaller s_{khj} , reduces the product scope of an exporter and thus increases the sales of a good closer to the core competence compared to those of a good farther away from the core.

Price Pass-Through

Let us verify that the pass-through of prices, keeping the aggregate set of varieties constant, is decreasing in the firm's market share. Prices, scope and market share can be written as functions of the aggregate mass of varieties Δ_j in country j .

$$\begin{aligned}\delta_{kij}^\theta &= \left[\frac{\theta + 2}{\theta \bar{q} w_i c_{ki} \tau_{ij}} \right] \frac{y_j}{\Delta_j} (1 - s_{kij}) \\ s_{kij} &= \frac{\delta_{kij}}{\Delta_j} \\ p_{kij}(\omega) &= \frac{w_i c_{ki} \tau_{ij} \omega^\theta}{1 - s_{kij}} \left(\frac{\delta_{kij}}{\omega} \right)^{\frac{\theta}{2}}\end{aligned}$$

Taking the total log derivative of the three equations, keeping Δ_j constant, yields the following price pass-through:

$$\frac{d \ln p_{kij}(\omega)}{d \ln \tau_{ij}} = \frac{1}{2} - \frac{s_{kij}}{2(\theta - \theta s_{kij} + s_{kij})}$$

which is decreasing in the firm's market share as documented by [Amiti et al. \(2014\)](#)³.

Price and Scope Elasticities

Finally, let us derive the price and scope elasticities with respect to trade costs and income of the destination, allowing Δ_j to change. Let us consider the elasticity of the product scope ϵ_δ with respect to trade costs. ϵ_δ is defined as the partial elasticity of exporter scope δ_{ij} with respect to bilateral trade costs τ_{ij} (or to firm's productivity or wages in i), keeping all other firms' decisions constant:

$$\epsilon_\delta = \frac{d \ln \delta_{kij}}{d \ln \tau_{ij}} = -\frac{1}{\theta + 2s_{kij}}$$

A reduction in τ_{ij} increases the product scope of exporters. Due to cannibalization effects, $|\epsilon_\delta|$ decreases with the firm's market share: larger firms are less reactive to changes in trade costs. In fact, a reduction in τ_{ij} increases the market share of exporters, who now face stronger cannibalization effects.

The elasticity of scope, ξ_δ , with respect to the destination's per capita income (or real exchange rate) equals:

$$\xi_\delta = \frac{d \ln \delta_{kij}}{d \ln y_j} = -\epsilon_\delta = \frac{1}{\theta + 2s_{kij}} > 0$$

³[Amiti et al. \(2014\)](#) compute the pass-through keeping the price index constant. In the baseline model, the equivalent exercise consists in computing the pass-through keeping the mass of varieties available to consumers constant. Moreover, in Bertrand competition, the pass-through with respect to the real exchange rate, allowing the mass of varieties available to consumers to vary, exhibits a U-shaped relationship with the firm market share, as documented by [Auer and Schoenle \(2016\)](#)

In partial equilibrium, the product scope of a firm rises with the per capita income of the destination. As other firms' choices are fixed, the exporter's market share also rises, strengthening the cannibalization effects. As a result, the stronger the cannibalization effects a firm faces, the less reactive it is to an increase in the per capita income of the destination.

The optimal price of a variety ω is:

$$p_{kij}(\omega) = \left(\frac{(\theta + 2)c_{kij}w_i\tau_{ij}}{\theta\bar{q}} \right)^{\frac{\theta}{2(\theta+1)}} \omega^{\frac{\theta}{2}} y_j^{\frac{\theta}{2(\theta+1)}} s_{kij}^{\frac{\theta}{2(\theta+1)}} (1 - s_{kij})^{-\frac{\theta+2}{2(\theta+1)}} \quad (9)$$

The expression is similar to that of [Simonovska \(2015\)](#), whose model considers single product firms. While in [Simonovska \(2015\)](#) prices depend on the aggregate export share of country i in country j 's total expenditure, here prices are an increasing function of the firm's market share. The pass-through of prices with respect to trade costs (ϵ_p) and per capita income ξ_p is:

$$\begin{aligned} \epsilon_p &= \frac{d \ln p_{kij}(\omega)}{d \ln \tau_{ij}} = \frac{1}{2} \\ \xi_p &= \frac{d \ln p_{kij}(\omega)}{d \ln y_j} = 1 - \epsilon_p = \frac{1}{2} \end{aligned}$$

Surprisingly, Cournot competition generates a constant elasticity of price with respect to trade costs⁴. The value of such an elasticity is the same arising from a model of monopolistic competition with Stone-Geary preferences ([Simonovska, 2015](#)). The markup elasticity is also a constant equal to 1/2, and thus the sales-weighted average markup elasticity (ρ) is 1/2. In partial equilibrium, the prices of a firm rise with the per capita income of the destination.

1.3 Extensions to the Baseline Model

In this section, I discuss in details the extensions to the model discussed in the paper.

1.3.1 Bertrand Competition

Let us consider the optimal scope and prices of multiproduct firms under Bertrand competition. Both Bertrand and Cournot competition yield qualitatively similar predictions. However, the relationship between prices, product scope and market shares is quantitatively different and such difference maps into different welfare gains from trade.

Solving the consumer problem yields the following direct demand of variety ω from firm k :

$$q_{kij}(\omega) = \frac{1}{\lambda_j p_{kij}(\omega)} - \bar{q}$$

⁴Results are different in Bertrand competition, in which the pass-through is U-shaped with respect to the market share of the firm, in line with the findings of [Auer and Schoenle \(2016\)](#).

Using the first order condition of the consumer problem in the budget constraint yields an expression for our marginal utility of income:

$$\lambda_j = \frac{\sum_{k=1}^M \delta_{kij}}{y_j + \bar{q}P_j}$$

where $P_j = \sum_{i=h,f} \sum_{k=1}^{M_i} \int_0^{\delta_{kij}} p_{kij}(\omega)$ is a price index.

Firm k chooses its prices $p_{kij}(\omega)$ for $\omega \in [0, \delta_{kij}]$, and mass of varieties δ_{kij} in order to maximize profits Π_k , taking other firms' choices as given. As in the baseline model of the paper, firms add new varieties until the demand for the last variety is zero. The following implicit equation defines the optimal mass of varieties supplied by the firm:

$$c_k(\delta_{khj}) = \frac{(1 - \bar{\mu}_{khj} s_{khj})}{\bar{q}\lambda_j}$$

where s_{khj} is the market share of firm k and $\bar{\mu}_{khj} = \frac{1}{\delta_{khj}} \int_0^{\delta_{khj}} \frac{p_{khj}(\omega) - c_{khj}(\omega)}{p_{khj}(\omega)}$ is the Lerner index of market power averaged on each product. Since the Lerner index is always less or equal to one, all else constant, cannibalization effects are stronger in Cournot competition.

Optimal prices are given by the following expression:

$$p_{khj}(\omega) = \underbrace{\frac{1}{(1 - \bar{\mu}_{khj} s_{khj})} \left(\frac{c_{khj}(\delta_{khj})}{c_{khj}(\omega)} \right)^{\frac{1}{2}}}_{\text{Mark-up}} c_{khj}(\omega)$$

The pricing equation is similar to the Cournot pricing rule: the largest firms charge the largest markups, and within a firm the price of products close to the core competence have the largest markup.

Using the functional form for marginal costs that we employed before, we can find the following expression for the optimal scope of firms:

$$\delta_{khj}^B = \left[\frac{(\theta + 2)^2}{\bar{q}\theta c_k y_i \tau_{ij}} \right]^{\frac{1}{\theta+1}} y_j^{\frac{1}{\theta+1}} \left[\frac{s_{khj}(1 - s_{khj})}{\theta + 2 - 2s_{khj}} \right]^{\frac{1}{\theta+1}}$$

Since cannibalization effects are weaker in Bertrand competition than in Cournot, all else constant, the product scope of exporters is larger when firms compete choosing their prices relative to quantities. In addition, the largest scope is reached at a market share equal to $s^* = 0.5\theta + 1 - [(0.5\theta + 1)0.5\theta]^{\frac{1}{2}}$. While in Cournot competition the maximum scope is reached at a market share of 50%, in Bertrand $s^* > 50\%$ and $s^* \rightarrow 50\%$ for $\theta \rightarrow \infty$.

The elasticity of scope and prices with respect to trade costs, keeping all other firms' decisions

constant, and are given by:

$$\begin{aligned}\epsilon_{\delta}^B &= \frac{d \ln \delta_{kij}}{d \ln \tau_{ij}} = -\frac{\theta + 2 - 2s_{kij}}{\theta^2 + 2\theta + 2(s_{kij} - s_{kij}^2)} \\ \epsilon_p^B &= \frac{d \ln p_{ij}^k}{d \ln \tau_{ij}} = \left[1 - \frac{\theta^2 + 2\theta}{2(\theta^2 + 2\theta) + 4(s_{ij} - s_{ij}^2)} \right]\end{aligned}$$

An increase in τ reduces the product scope of exporters. Due to cannibalization effects, $|\epsilon_{\delta}|$ is decreasing in the firm's market share. Bertrand competition yields a non-monotone, hump-shaped relationship between price elasticity and firm's market share. For a market share of zero and one, the price elasticity collapses to a half. However, firms with an intermediate market share increase their prices by more than a half. The maximum pass through is reached at a market share of 50%. The hump-shape of the pass-through in Bertrand competition is similar to what predicted in other work by [Atkeson and Burstein \(2008\)](#). The authors use nested CES preferences and thus predict a full pass-through for firms with a market share of 0 or 1⁵.

The elasticity of scope, ξ_{δ} and of price ξ_p with respect to the destination's per capita income (or real exchange rate) equals:

$$\begin{aligned}\xi_{\delta} &= \frac{d \ln \delta_{kij}}{d \ln y_j} = -\epsilon_{\delta} > 0 \\ \xi_p &= \frac{d \ln p_{kij}(\omega)}{d \ln y_j} = 1 - \epsilon_p > 0\end{aligned}$$

The pass-through with respect to the real exchange rate in Bertrand competition is then U-shaped, consistently with the evidence that [Auer and Schoenle \(2016\)](#) provide.

Finally, let us consider our baseline model with Bertrand competition, in which the welfare formula equals:

$$d \ln W_j^{Bertrand} = \frac{(\theta + 2)\theta}{2(\theta + 1)} \left[1 + \frac{s_{jj}}{(1 - s_{jj})(\theta + 2 - 2s_{jj})} \right] (-d \ln s_{jj}) \quad (10)$$

Given a θ and a change in s_{jj} , $d \ln W_j^{Bertrand} < d \ln W_j^{Cournot}$. Due to the different demand elasticities faced by firms in the two types of competition, the aggregate set of varieties increases by less in Bertrand competition relative to Cournot after a trade liberalization.

⁵A firm with a market share of 0 is monopolistically competitive within the nest of products it belongs, while a firm with a market share of 1 is the only producer of a particular nest of products, but it is monopolistically competitive with respect the remaining product groups.

1.3.2 Luxuries and Necessities

Consider the following utility function:

$$U_h = \sum_{i=h,f} \sum_{k=1}^{M_i} \int_0^{\delta_{kih}} [\ln(q_{kih}(\omega) + \bar{q}(\omega)) - \ln \bar{q}(\omega)] d\omega$$

where $\bar{q}(\omega)$ is variety-specific. $\bar{q}(\omega)$ controls the intercept of the Engel curve: the higher the $\bar{q}(\omega)$, the lower the intercept. The aggregate inverse demand function and marginal utility of income are analogous to the baseline model.

The firm's problem is identical to the baseline case with the exception that each firm pays a constant marginal cost of production and delivery c_{kij} across all its varieties. Firm's varieties are indexed by $\omega \in [0, \delta_{kij}]$, and $\bar{q}(\omega)$ is increasing in ω . The first order condition of the firm's problem with respect to the mass of varieties δ_{kij} yields the following implicit solution for the optimal scope:

$$\bar{q}(\delta_{kij}) = \frac{1 - s_{kij}}{\lambda_j c_{kij}}$$

The price set by firm k from i to j equals:

$$p_{kij}(\omega) = \frac{1}{1 - s_{kij}} \left(\frac{\bar{q}(\delta_{kij})}{\bar{q}(\omega)} \right)^{\frac{1}{2}} c_{kij}$$

On the other hand, revenues are equal to:

$$r_{kij} = \frac{L_j}{\lambda_j} \left[1 - \left(\frac{\bar{q}(\omega)}{\bar{q}(\delta_{kij})} \right)^{\frac{1}{2}} \right]$$

As in the core competence environment, the closer a variety is to the core, the higher are markups and revenues. However, there is a difference between the baseline model and the one here presented. In the former there is a negative relationship between prices and revenues: the core goods have low prices but high revenues. On the other hand, in this section, where product heterogeneity within a firm is generated through differences in consumers' preferences the relationship between sales and prices is positive.

Finally, if we assume that $\bar{q}(\omega) = \bar{q}\omega^\theta$, the scope of an exporter is identical to the baseline case:

$$\delta_{kij} = \left[\frac{\theta + 2}{\theta \tau_{ij} y_i c_{ki}} s_{kij} (1 - s_{kij}) y_j \right]^{\frac{1}{\theta+1}}$$

As stated by [Bernard et al. \(2011\)](#), whether the heterogeneity within firms is generated by a technological assumption or a preference assumption, results are identical.

1.3.3 Brand Differentiation

In this section, I show how we can introduce brand differentiation by modifying the standard Stone-Geary preferences. In the baseline model, introducing a new variety reduces the sales of a firm regardless of whether the variety is introduced by the same firm or by another. In other words, the competition between goods within a firm is the same as the competition between goods across firms. In this extension, I break this relationship, modeling brand differentiation and therefore differentiating between the effect of a firm introducing new variety on its own sales from existing varieties, and on other firms sales.

Introducing brand differentiation is usually achieved with a nested CES framework as in [Atkeson and Burstein \(2008\)](#) for single product firms and in [Hottman et al. \(2016\)](#) for multiproduct firms. However, nesting a non-homothetic function into a CES aggregator is highly intractable. Another way of introducing brand differentiation in non-homothetic preferences is that of [Dhingra \(2013\)](#), who introduces to a linear-quadratic preference the sum of quadratic firm's quantity aggregates. As a result, the inverse demand function of one variety not only depends on the quantity of that variety, but also on the quantity index of the same firm. Therefore, raising the output of one variety has a direct effect on the demand for all the firm's existing varieties. Unfortunately, a similar approach is not as tractable when applied to the Stone-Geary case.

Inspired by [Dhingra \(2013\)](#), I add a new term to the Stone-Geary preferences so that the firm's scope directly affects the indirect demand of all existing varieties, in addition to its indirect effect through market aggregates. In particular I consider the following utility function:

$$U_j = \sum_{i=h,f} \sum_{k=1}^M \int_0^{\delta_{kij}} \left[\ln \left(\frac{q_{kij}(\omega)}{\delta_{kij}^\gamma} + \bar{q} \right) - \ln \bar{q} \right] d\omega \quad (11)$$

The quantity produced by each variety is weighted by the number of varieties of the firm. This specification is similar in spirit to those that introduce product quality as a weight on the quantities in the utility function ([Manova and Zhang, 2012](#); [Eckel et al., 2015](#)). If $\gamma > 0$, the larger the scope of a firm, the smaller the utility from an additional quantity consumed. The aggregate inverse demand function arising from (11) is:

$$p_{kij}(\omega) = \frac{L_j}{\lambda_j} \left(\frac{1}{x_{kij}(\omega) + \bar{q} L_j \delta_{kij}^\gamma} \right)$$

and the marginal utility of income λ_j is:

$$\lambda_j = \frac{1}{y_j} \sum_{i=h,f} \sum_{k=1}^{M_i} \int_0^{\delta_{kij}} \frac{x_{kij}(\omega)}{x_{kij}(\omega) + \bar{q} L_j \delta_{kij}^\gamma} d\omega$$

The elasticity of prices with respect to the scope of the firm is then:

$$\frac{d \ln p_{kij}(\omega)}{d \ln \delta_{kij}} = -\frac{d \ln \lambda_j}{d \ln \delta_{kij}} - \gamma \frac{\bar{q} L_j \delta_{kij}^\gamma}{x_{kij} + \bar{q} L_j \delta_{kij}^\gamma}$$

If $\gamma = 0$, the extension collapses to the baseline model, in which a scope expansion reduces the inverse demand function only through its effects on the marginal utility of income λ . If $\gamma > 0$, an expansion in the scope further reduces the demand of the firm that introduced the new varieties. Moreover, a change in the scope of firm $k' \neq k$ on the prices of firm k only depends on λ_j .

The first order conditions are:

$$\begin{aligned} \{x_{kij}(\omega)\} : \frac{L_j}{\lambda_j} (1 - s_{kij}) \frac{\bar{q} L_j \delta_{kij}^\gamma}{(x_{kij} + \bar{q} L_j \delta_{kij}^\gamma)^2} &= c_{kij}(\omega) \\ \{\delta_{kij}\} : \frac{L_j}{\lambda_j} (1 - s_{kij}) \left[\frac{x_{kij}(\delta_{kij})}{x_{kij}(\delta_{kij}) + \bar{q} L_j \delta_{kij}^\gamma} - \gamma \bar{q} L_j \delta_{kij}^{\gamma-1} \int_0^{\delta_{kij}} \frac{x_{kij}(\omega)}{(x_{kij}(\omega) + \bar{q} L_j \delta_{kij}^\gamma)^2} \right] &= c_{kij}(\delta_{kij}) x_{kij}(\delta_{kij}) \end{aligned}$$

Using the first FOC, and setting the quantity demanded to zero, yields the cost cutoff \bar{c}_{kij} for firm k in destination j :

$$\bar{c}_{kij} = \frac{1 - s_{kij}}{\bar{q} \lambda_j \delta_{kij}^\gamma}$$

The optimal quantity supplied by a firm is:

$$x_{kij}(\omega) = \bar{q} L_j \delta_{kij}^\gamma \left[\left(\frac{\bar{c}_{kij}}{c_{kij}(\omega)} \right)^{\frac{1}{2}} - 1 \right] \quad (12)$$

Analogously to the extension on diseconomies of scope shown later, the marginal cost of the last variety is a constant fraction of the choke marginal cost: $c_{kij}(\delta_{kij}) = \Phi \bar{c}_{kij}$.

$$c_{kij}(\delta_{kij}) = \Phi \bar{c}_{kij}$$

where Φ is implicitly defined by:

$$(1 - \Phi^{\frac{1}{2}})^2 = \gamma \left(1 - \frac{\Phi}{\theta + 1} \right) \quad (13)$$

The new preferences only affect qualitatively the scope of the firm. Since $\Phi \leq 1$, the scope is smaller than the one predicted by the previous model. Moreover, the elasticity of scope with respect to per capita income additionally depends on γ .

$$\delta_{kij} = \left(\frac{\Phi(\theta + 2)}{\theta \tau_{ij} c_{ki} w_i} y_j s_{kij} (1 - s_{kij}) \right)^{\frac{1}{\theta + \gamma + 1}} \quad (14)$$

The welfare gains from trade are given by the following formula:

$$d \ln W_j = \frac{\tilde{\kappa}}{\epsilon} \left[1 - \frac{\rho}{\epsilon + 1} \right] \left[1 + \frac{\epsilon s_{jj}}{1 - s_{jj}} \right] (-d \ln s_{jj}) \quad (15)$$

where $\epsilon = \theta + \gamma$, and $\tilde{\kappa}$ is a constant that depends of Φ . The formula is almost identical to the baseline one. In particular, the mismeasurement due to ignoring cannibalization effects still depends on ϵ and s_{jj} .

Similar results can be obtained with a different preference specification:

$$U_j = \sum_{k=1}^M \delta_{kij}^\gamma \int_0^{\delta_{kij}} \ln(q_{kij}(\omega) + \bar{q}) d\omega \quad (16)$$

For $\gamma = 0$, the utility function is the same as the baseline model. The parameter γ controls the degree of brand differentiation, or the consumer's love for the variety of one firm. I restrict $\gamma > -1$ ⁶. For a positive γ consumers demand larger quantities from wider brands, and vice versa for a negative value of γ .

These preferences are similar to those of [Benassy \(1996\)](#), [Blanchard and Giavazzi \(2003\)](#) and [Eckel \(2008\)](#), where a standard CES aggregate is multiplied by a power function of the number of varieties⁷. The purpose of these authors is to break the relationship between consumers' love for variety and firms' market power in the CES framework.

1.3.4 Fixed Cost per Variety

Suppose that firms must pay a fixed cost $f_{k,ij}(\omega)$ per variety, and that $f_{k,ij}(\omega)$ is weakly increasing in ω . A firm introduces varieties until the profits from the last variety, discounted by the cannibalization effects, barely cover the fixed cost per variety. The introduction of a fixed cost generates a positive relationship between the product scope of the firm and size of the destination: larger markets yield higher revenues that can cover a larger fixed cost. A fixed cost of entry per variety replicates, at the firm level, what [Eaton et al. \(2011\)](#) achieved at the aggregate level. The authors introduced a fixed cost of entry per firm to rationalize the positive relationship between extensive margin of trade and size of the destination.

To further clarify the role played by the fixed cost of entry, consider a scenario in which marginal costs $c_{k,ij}(\omega)$ are zero for all varieties. Per capita income and size of the destination have then identical effects on the scope of firms: $\delta_{k,ij} f(\delta_{k,ij}) = s_{k,ij}(1 - s_{k,ij})y_j L_j$. To summarize, fixed costs per variety generate a positive relationship between scope and size of the destination whereas non-homothetic preferences and the core competence assumption yield a positive relationship between

⁶The limit for $\delta_{kij} \rightarrow 0$ of the utility function depends on the limit of $\delta_{kij}^\gamma \int_0^{\delta_{kij}} \ln(q_{kij}(\omega) + \bar{q}) d\omega$. For $\gamma > -1$, using de l'Hopital rule, the limit is 0.

⁷The utility function of [Benassy \(1996\)](#) is: $U = n^{\nu+1-\frac{1}{\theta}} \left(\sum_{i=1}^n q_i^\theta \right)^{\frac{1}{\theta}}$, where ν controls the love for variety of consumers. Such extension is also present in the working paper version of Dixit-Stiglitz

scope and per capita income of the destination.

1.3.5 Multiple Product Lines

Let us now consider a case in which firms produce different product lines indexed by $n_{kij} \in [0, N_{kij}]$. Each product line n requires a fixed cost in domestic labor units $F_{kij}(n)$. Within each product line n a firm produces a continuum of varieties indexed by $\omega \in [0, \delta_{kij}(n)]$. Within each product line, firms have a core competence, and additional varieties are produced at increasing marginal cost. Product lines differ in terms of their fixed cost of production: $F_{kij}(n)$ is increasing in n . Firms introduce product lines that require the lowest fixed cost first, and then introduce lines with higher fixed costs. An alternative structure, that yields the same predictions, assumes that the $\bar{q}(n)$ varies across product lines.

Similarly to the baseline model, the number of varieties $\delta_{kij}(n)$ within each product line n is increasing in the per capita income of the destination and it features a hump-shaped relationship with respect to the market share of the firm in the destination. There is a non-monotone hump-shaped relationship between the number of product lines N_{kij} and the market share of the firm as well. The effect of market size is twofold. Firms are, in fact, facing a new tradeoff: either export the core varieties of several product line, which requires a larger fixed cost, or export fewer and longer product lines, with many non-core varieties. In larger destinations, the fixed cost of entry have a smaller impact and firms export a larger number of shorter product lines. On the other hand, firms export fewer but longer product lines in smaller economies. Overall, the sum of varieties exported by a firm across its product lines is increasing in size and per capita income of the destination.

1.3.6 Diseconomies of Scope

I consider the following expression for the marginal cost of production and delivery of a variety ω :

$$\text{Marginal cost}_{kij}(\omega) = \delta_{kij}^\gamma c_{kij}(\omega) \tag{17}$$

Similarly to the baseline model I assume that firms technology is characterized by a core competence and that $c_{kij}(\omega)$ is increasing in ω .

In addition, the marginal cost of each variety depends on the scope of the firm. If $\gamma > 0$, the firm technology exhibits diseconomies of scope: the same variety ω is produced at a higher marginal cost if the product scope expands, as in [Nocke and Yeaple \(2014\)](#)⁸. This captures potential inefficiencies arising from managing an increasing number of product lines. Vice versa if

⁸[Nocke and Yeaple \(2014\)](#) assume that firms have to allocate their organizational capital to the varieties produced. Firms first choose their organizational capital and then decide the optimal number of varieties. As the scope increases, firms have to allocate a scarce resource, the organizational capital, to an increasing number of varieties, thus increasing the marginal cost of each variety.

$\gamma < 0$, firms technology exhibits economies of scope: the marginal cost of producing a variety falls with the scope. I consider a closed economy here for ease of notation.

Firms maximize their profits choosing quantities (in Cournot) or prices (in Bertrand) and the mass of varieties they produce (in both). By the first order condition with respect to quantities (in Cournot) or prices (in Bertrand) we can find a choke marginal cost denoted by $\delta_k^\gamma \bar{c}_k$. $\delta_k^\gamma \bar{c}_k$ is the marginal cost of a variety such that its demand is zero, or equivalently, its optimal price equals the choke price. The choke marginal costs are denoted by $\delta_k^\gamma \bar{c}_k^B$ in Bertrand competition and $\delta_k^\gamma \bar{c}_k^C$ in Cournot competition and are defined as:

$$\begin{aligned}\delta_k^\gamma \bar{c}_k^B &= \bar{p}_k(1 - \bar{\mu}_k s_k) \\ \delta_k^\gamma \bar{c}_k^C &= \bar{p}_k(1 - s_k)\end{aligned}$$

where s_k is the market share of firm k and $\bar{\mu}_k = \frac{1}{\delta_k} \int_0^{\delta_k} \frac{p_k(\omega) - \delta_k^\gamma c_k(\omega)}{p_k(\omega)} d\omega$ is the average Lerner index of market power per product. Since the Lerner index is always less than 1, $\bar{c}_k^B > \bar{c}_k^C$.

The first order conditions with respect to the mass of varieties implicitly define the optimal scope produced by each firm by determining the marginal cost for the last variety $c_k(\delta_k)$. In my baseline model, when $\gamma = 0$, firms introduce varieties until the last variety has a marginal cost equal to the “choke” marginal cost. Intuitively, if $\gamma < 0$ the firm enjoys economies of scope and has incentives to introduce more varieties, but the decision is bounded by the choke marginal cost. On the other hand, with diseconomies of scope ($\gamma > 0$) the firm restricts its product scope, so that the marginal cost of the last variety is a fraction of the choke marginal cost. Because introducing a new variety increases the marginal cost of all existing varieties, the firm has incentives to limit its scope expansion.

The optimal mass of varieties is such that the marginal cost of the last variety is a fraction Φ_k of the choke marginal cost:

$$\begin{aligned}c_k(\delta_k) &= \Phi_k \bar{c}_k \quad \text{where} \\ \Phi_k &= 1 && \text{if } \gamma \leq 0 \\ \Phi_k &< 1 && \text{if } \gamma > 0\end{aligned}$$

I use the baseline functional form for the marginal cost of the firm: $\delta_k^\gamma c_k(\omega) = \delta_k^\gamma y c \omega^\theta$, where c is a cost parameter, y is the wage and θ controls how fast marginal costs rise as the firm introduces varieties far from its core competence. I can prove that Φ_k is a constant that depends uniquely on the technology parameters θ and γ . In addition, Φ is the same in both types of competition. The fact that Φ is a constant makes this extension to the model to be extremely tractable.

Solving the model allows me to find the definition of market share, which is identical in both

types of competition:

$$s_k^B = s_k^C = \frac{\delta_k}{\sum_{j=1}^M \delta_j} \quad (18)$$

Finally, the product scope of firms as a function of the market share of the firm itself equals:

$$\delta_k^B = \left[\frac{\theta + 2}{\bar{q}\theta c} \left(\frac{s_k - s_k^2}{1 - \kappa_1 s_k} \right) \right]^{\frac{1}{\theta + \gamma + 1}}$$

$$\delta_k^C = \left[\frac{\Phi}{\bar{q}(1 - \kappa_1)c} (s_k - s_k^2) \right]^{\frac{1}{\theta + \gamma + 1}}$$

where $\kappa_1 = \frac{\Phi^{\frac{1}{2}}}{0.5\theta + 1}$. Since we are considering a closed economy, per capita income disappears from the equation.

In both cases, there is a hump-shaped relationship between market share of the firm and product scope of the firm. The market share at which the maximum scope is attained is:

$$s_k^{*B} = \frac{1 - \sqrt{1 - \kappa_1}}{\kappa_1}$$

$$s_k^{*C} = \frac{1}{2}$$

While in Cournot the maximum scope is reached at a market share of 0.5, no matter the values of θ and γ , Bertrand competition allows a more flexible analysis of the cannibalization effects. s_k^{*B} both depends on θ and γ and, as in [Feenstra and Ma \(2007\)](#), $s_k^{*B} \geq 0.5$. In particular, s_k^{*B} is increasing in $\kappa_1 = \frac{\Phi^{\frac{1}{2}}}{0.5\theta + 1}$. If the effect of diseconomies of scope is large (larger γ), firms limit their product scope expansion (small Φ). Hence, the highest scope is reached at a smaller market share s_k^{*B} if γ is large.

1.4 Welfare: Local Approximation VS General Formula

In this subsection, I study how the local approximation to the welfare gains from trade compares to the general formula. It is useful to re-write the two formulas here after:

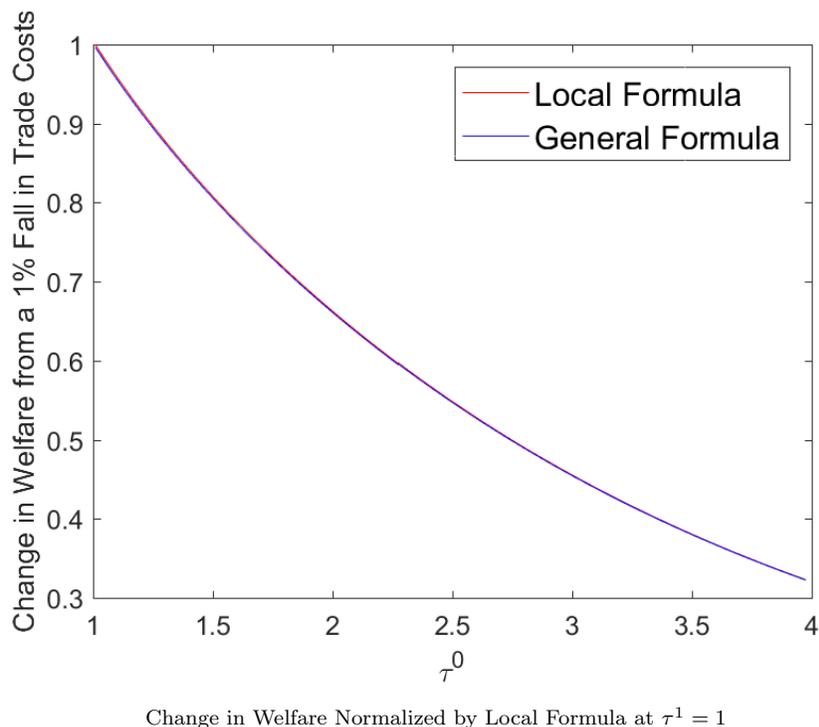
$$d \ln W = \frac{\theta(\theta + 2)}{2(\theta + 1)} \left[1 + \frac{s^0}{\theta(1 - s^0)} \right] (-d \ln s)$$

$$\ln \hat{W} = \ln \left[\frac{\theta + 2}{\theta} \left[\hat{s} \left(\frac{1 - s^0}{1 - \hat{s}s^0} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta^2}{2(\theta + 1)}} - \frac{2}{\theta} \right]$$

I consider a symmetric country model, which depends on a vector of four parameters $\Theta = [\tau, \theta, L, F]$. Without loss of generality, I choose the following parametrization: $\theta = 0.5$, $L = 1$ and $F = 0.32$, so that in the case of costless trade, the market share of the typical firm is 0.3. I

let τ vary in $[1, 4]$, with 1% steps. The results are robust to alternative choices of the vector of parameters Θ . Starting from $\tau = 4$, I compute the welfare gains from trade by reducing τ of 1 percentage point, until $\tau = 1$. For a change in τ from τ^0 to τ^1 , I compute the corresponding change in the market share $d \ln s = \ln(\hat{s}) = \ln\left(\frac{s^1}{s^0}\right)$. I then use the local and general formula to compute the welfare gains from trade. Figure 1 shows that the local approximation is virtually identical to the general formula: the local approximation slightly overestimates the change in welfare for small trade costs, but becomes identical to the general formula for larger trade costs.

Figure 1: Welfare: Local and General Formula



1.5 Integer Number of Firms

In this section, I consider an extension to the baseline symmetric country model, in which I take into account the integer problem. In the baseline model, free entry drives profits to zero and per capita income y equals the wage rate w . For profits to be equal zero, the number of firms M must be a continuous variable — hence, the integer problem. In this section I restrict the number of firms M to defined over the set of integer numbers. Following [Eaton et al. \(2012\)](#), the equilibrium number of firms M^{eq} is attained when profits are positive at $M = M^{eq}$, and profits are negative at $M = M^{eq} + 1$. As a result of this assumption, total profits are positive and, therefore, per capita income and wage differ. Such a difference has an effect on the total mass of varieties available for consumption, and on welfare.

I normalize the wage rate to one, and let y be determined endogenously. In particular, per

capita income equals the sum of the unit wage rate and the per capita profits:

$$y = 1 + \frac{\Pi M}{L} \quad (19)$$

For a given value of M , the market clearing condition (??) and the relationship between domestic and foreign market shares (??) pin down s and s^* :

$$s + s^* = \frac{1}{M} \quad (20)$$

$$\frac{s^\theta}{1-s} = \tau \frac{s^{*\theta}}{1-s^*} \quad (21)$$

For a given value of M , firm's profits are:

$$\Pi(M) = yL \left[\frac{s^2 + \theta s}{\theta + 1} + \frac{s^{*2} + \theta s^*}{\theta + 1} \right] - F$$

Using (19), profits are given by:

$$\Pi(M) = \frac{L \left[\frac{s^2 + \theta s}{\theta + 1} + \frac{s^{*2} + \theta s^*}{\theta + 1} \right] - F}{1 - M \left[\frac{s^2 + \theta s}{\theta + 1} + \frac{s^{*2} + \theta s^*}{\theta + 1} \right]} \quad (22)$$

The equilibrium is a vector of market shares $[s, s^*]$, number of firms M , and per capita income y , which satisfies (20), (21), and (19). Moreover $\Pi(M) \geq 0$ and $\Pi(M + 1) < 0$

What are the implications for welfare of the assumption of a finite number of firms? First, consider the mass of varieties available for consumption, which equals:

$$\Delta = \left[\frac{\theta + 2}{\bar{q}c\theta} y(1-s)s^{-\theta} \right]^{\frac{1}{\theta+1}}$$

Therefore, the change in the indirect utility due to a change in trade costs can be expressed as a function of the change in s and y :

$$d \ln V = d \ln \Delta = \frac{\theta}{\theta + 1} \left(1 + \frac{s}{\theta(1-s)} \right) (-d \ln s) + \frac{\theta}{\theta + 1} (d \ln y)$$

The equivalent variation in income for a small variation of trade costs can be expressed as:

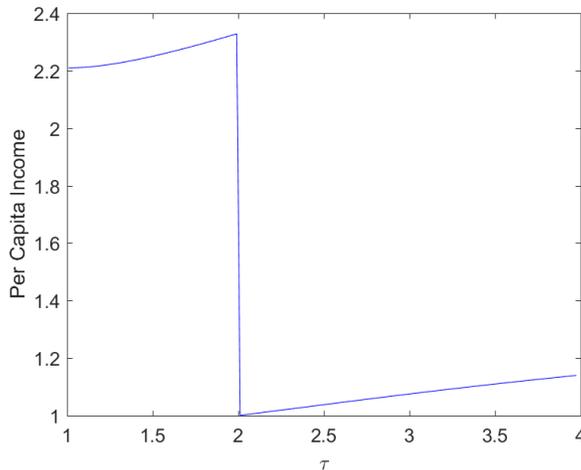
$$d \ln W = \underbrace{\frac{\theta(\theta + 2)}{2(\theta + 1)} \left[1 + \frac{s}{\theta(1-s)} \right]}_{\text{Baseline Formula}} (-d \ln s) + \underbrace{\frac{\theta(\theta + 2)}{2(\theta + 1)}}_{\text{Integer Problem}} d \ln y \quad (23)$$

To assess the degree with which ignoring the integer problem biases the baseline formula I use the following algorithm. Without loss of generality, I choose the following parametrization:

$\theta = 0.5$, $L = 1$ and $F = 0.26$. I let τ vary in $[1, 4]$, with 1% steps. The algorithm finds the equilibrium values for $[s, s^*, M, y]$. I start at $M = 1$. Given M , I find $[s, s^*]$ using (20) and (21). I then compute $\Pi(M)$ using (22) and y using (19). Then, I increase the number of firms by one unit and repeat the procedure. The algorithm stops at a value of M such that $\Pi(M) \geq 0$ and $\Pi(M + 1) < 0$. I find the equilibrium for $\tau \in [1, 4]$ and compare the baseline welfare formula to (23).

Figure 2 shows the relationship between per capita income and iceberg trade costs. When larger trade costs leave unchanged the number of firms, (nominal) per capita income rises as trade costs increase. Larger trade costs increase the consumption of domestic produced varieties, which are sold at higher markups than the exported varieties. As a result, larger trade costs increase firms' profits and, therefore, per capita income. As trade costs increase further, firms' profits become large enough to allow for additional entry. As firms enter, profits decline and per capita income falls steeply.

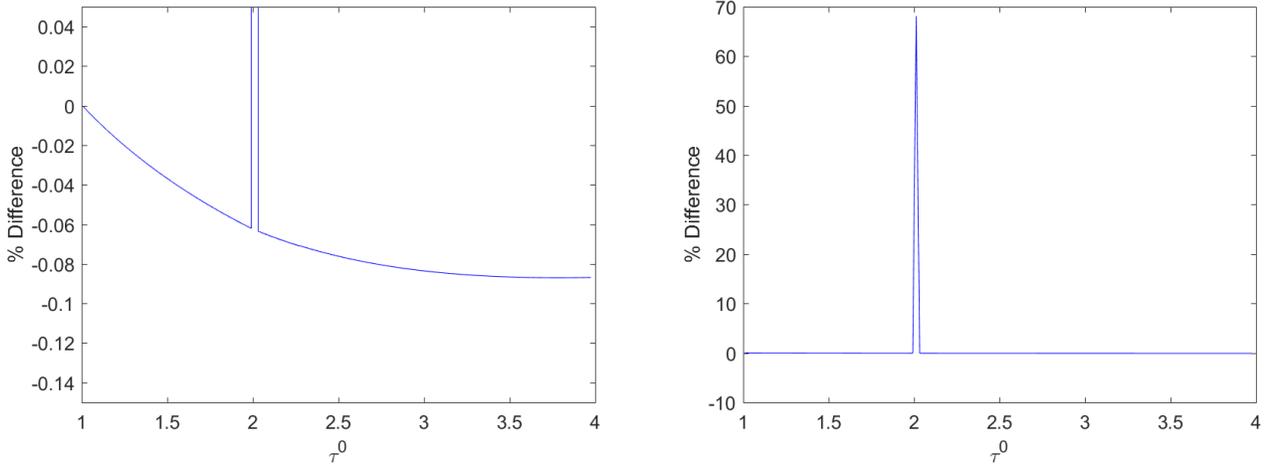
Figure 2: Per Capita Income



The difference in the welfare predicted by the baseline model and a model with an integer number of firms follows directly from figure 2. A reduction in trade costs that leaves the number of firms unchanged causes a reduction in per capita income. Thus, a model with an integer number of firms generates smaller gains relative to the baseline model when trade leaves M unchanged⁹. On the other hand, a reduction in trade costs that reduces the number of firms, causes nominal per capita income to rise. Hence, a model with an integer number of firms generates larger gains relative to the baseline model when trade changes M . Figure 3 illustrates the difference in the gains from trade between the model illustrated in this section and my baseline.

⁹Notice that the gains from trade predicted by a model with an integer number of firms are still larger than those predicted by ACDR.

Figure 3: Welfare Gains: Baseline Model VS Model with Integer Number of Firms



The plot shows $100 \left(\frac{\hat{W}_I}{\hat{W}_{Baseline}} - 1 \right)$, where $\hat{W} = 1 + d \ln W$, and subscript I denotes the model with an integer number of firms.

1.6 Competitive Fringe

This section describes in details a model where large multiproduct firms coexist with a competitive fringe. There is a discrete number M_i of firms, each producing a continuum of varieties indexed by $\omega \in [0, \delta_{kij}^o]$. Superscript o denotes large multiproduct firms (oligopolists). Each firm pays a fixed cost F_i , and maximizes its profits taking other oligopolists' and the competitive fringe's decisions as given. As in the baseline model, the marginal costs of production and delivery of a variety ω is $c_{kij}^o(\omega) = \tau_{ij} w_i c_{ki}^o \omega^\theta$.

There is also an infinite number of perfectly competitive firms in each country. Let superscript c denote the variables of interest of the competitive fringe. The competitive fringe in i sells to j a continuum of varieties indexed by $\omega \in [0, \delta_{ij}^c]$. The marginal cost of producing a variety ω by the competitive fringe is $c_{ij}^c(\omega) = \tau_{ij} w_i c_i^c \omega^\theta$. Without loss of generality I assume that θ , the elasticity of marginal costs with respect to the distance from the core, is the same as the large firms. However, large and small firms differ by their cost parameter and $c_i^c > c_{ki}^o$. Both superstars and the competitive fringe simultaneously choose scope and quantities.

Let us assume that preferences of consumers in country j can be represented by the following utility function:

$$U_j = \sum_{i=h,f} \left(\alpha \sum_{k=1}^{M_i} \int_0^{\delta_{kij}^o} [\ln(q_{kij}^o(\omega) + \bar{q}) - \ln(\bar{q})] d\omega + (1 - \alpha) \int_0^{\delta_{ij}^c} [\ln(q_{ij}^c(\omega) + \bar{q}) - \ln(\bar{q})] d\omega \right)$$

where α is the weight on the goods produced by large multiproduct firms. If $\alpha = 1$, the model becomes the baseline model, whereas if $\alpha = 0$, all varieties demanded are produced by the competitive fringe.

Solving the consumer's problem, and aggregating across consumers yield the following inverse demand functions for the goods produced by superstars and the competitive fringe:

$$p_{kij}^o(\omega) = \frac{\alpha L_j}{\lambda_j(x_{kij}^o(\omega) + \bar{q}L_j)} \quad p_{ij}^c(\omega) = \frac{(1 - \alpha)L_j}{\lambda_j(x_{ij}^c(\omega) + \bar{q}L_j)} \quad (24)$$

where the marginal utility of income λ_j equals:

$$\lambda_j = \frac{1}{y_j} \sum_i \left[\alpha \sum_{k=1}^{M_i} \int_0^{\delta_{kij}^o} \frac{x_{kij}^o(\omega)}{x_{kij}^o(\omega) + \bar{q}L_j} + (1 - \alpha) \int_0^{\delta_{ij}^c} \frac{x_{ij}^c(\omega)}{x_{ij}^c(\omega) + \bar{q}L_j} \right] \quad (25)$$

Let us now turn to the problem of superstars, which is identical to the baseline case. Profit maximization yields the following solutions for the quantity of a variety ω and scope of firm k from i to j :

$$x_{kij}^o(\omega) = \bar{q}L_j \left[\left(\frac{\delta_{kij}^o}{\omega} \right)^{\frac{\theta}{2}} - 1 \right]$$

$$\tau_{ij} w_i c_{ki}^o (\delta_{kij}^o)^\theta = \frac{\alpha(1 - s_{kij}^o)}{\bar{q}\lambda_j}$$

The revenues of a superstar are proportional to its scope as in the baseline model:

$$r_{kij}^o = \alpha \frac{\theta L_j}{(\theta + 2)\lambda_j} \delta_{kij}^o \quad (26)$$

Let us now look at the competitive fringe. Firms' prices are equal to their marginal costs of production. The fringe adds new varieties until the last variety has zero demand. Thus, the optimal quantity and scope of the competitive fringe is:

$$x_{ij}^c(\omega) = \bar{q}L_j \left[\left(\frac{\delta_{ij}^c}{\omega} \right)^\theta - 1 \right]$$

$$\tau_{ij} w_i c_i^c (\delta_{ij}^c)^\theta = \frac{(1 - \alpha)}{\bar{q}\lambda_j}$$

The aggregate revenues of the competitive fringe are proportional to its scope as it is the case with superstars:

$$r_{ij}^c = (1 - \alpha) \frac{\theta L_j}{(\theta + 1)\lambda_j} \delta_{ij}^c \quad (27)$$

Let Δ_j^o and Δ_j^c be the aggregate mass of varieties supplied by large multiproduct firms and the competitive fringe to country j . Using (26) and (27), we can determine the market share of

superstar k from i to j , s_{kij}^o , and the market share of the competitive fringe from i to j , Λ_{ij}^c as:

$$s_{kij}^o = \frac{\frac{\alpha}{\theta+2} \delta_{kij}^o}{\frac{\alpha}{\theta+2} \Delta_j^o + \frac{1-\alpha}{\theta+1} \Delta_j^c} \quad \Lambda_{ij}^c = \frac{\frac{1-\alpha}{\theta+1} \delta_{ij}^c}{\frac{\alpha}{\theta+2} \Delta_j^o + \frac{1-\alpha}{\theta+1} \Delta_j^c}$$

Finally, the optimal scope of oligopolists are identical to the baseline case, while the optimal scope of the competitive fringe reflects the lack of cannibalization effects.

$$\delta_{kij}^o = \left[\frac{\theta+2}{\theta \bar{q} w_i c_{ki}^o \tau_{ij}} \right]^{\frac{1}{\theta+1}} y_j^{\frac{1}{\theta+1}} [s_{kij}^o (1 - s_{kij}^o)]^{\frac{1}{\theta+1}} \quad \delta_{ij}^c = \left[\frac{\theta+1}{\theta \bar{q} w_i c_i^c \tau_{ij}} \right]^{\frac{1}{\theta+1}} y_j^{\frac{1}{\theta+1}} (\Lambda_{ij}^c)^{\frac{1}{\theta+1}} \quad (28)$$

To derive the welfare formulas, it is convenient to introduce the following notation. Let μ_j^o be the expenditure share of consumers in j on goods produced by home and foreign superstars, and μ_j^c be the expenditure share on the competitive fringe goods:

$$\mu_j^o = \frac{\frac{\alpha}{\theta+2} \Delta_j^o}{\frac{\alpha}{\theta+2} \Delta_j^o + \frac{1-\alpha}{\theta+1} \Delta_j^c} \quad \mu_j^c = \frac{\frac{1-\alpha}{\theta+1} \Delta_j^c}{\frac{\alpha}{\theta+2} \Delta_j^o + \frac{1-\alpha}{\theta+1} \Delta_j^c} \quad (29)$$

Let \tilde{s}_{kij}^o be the ratio of the sales of an oligopolist k from i to j relative to the sales of all oligopolists in j . In other words, \tilde{s}_{kij}^o is the market share of k in the superstars' market in country j . Let $\tilde{\lambda}_{ij}^c$ be the share of the competitive fringe from i relative to the total sales of all competitive fringes in j . The relationship between \tilde{s}_{kij}^f , for $f = o, c$, and the number of varieties of superstars and competitive fringe available for consumption is given by

$$\tilde{s}_{kij}^o = \frac{s_{kij}^o}{\mu_j^o} = \frac{\delta_{kij}^o}{\Delta_j^o} \quad \tilde{s}_{kij}^c = \frac{\Lambda_{ij}^c}{\mu_j^c} = \frac{\delta_{ij}^c}{\Delta_j^c} \quad (30)$$

Let us now impose the symmetry assumption on superstars and drop subscript k . The zero profit condition for superstars is analogous to the baseline model. Without loss of generality, let us normalize the per capita income of country j to 1. Consider the domestic optimal scope of domestic superstars and competitive fringe (28). Plugging δ_{jj}^o and δ_{jj}^c into (30) yields the total mass of varieties produced by superstars and the competitive fringe in j :

$$\Delta_j^o = \left[\frac{\theta+2}{\theta \bar{q} c_j^o} \right]^{\frac{1}{\theta+1}} \mu_j^o (s_{jj}^o)^{-\frac{\theta}{\theta+1}} (1 - s_{jj}^o)^{\frac{1}{\theta+1}} \quad \Delta_j^c = \left[\frac{\theta+1}{\theta \bar{q} c_j^c} \right]^{\frac{1}{\theta+1}} \mu_j^c (\Lambda_{jj}^c)^{-\frac{\theta}{\theta+1}} \quad (31)$$

We can now derive the welfare formula. First, using the optimal quantities produced by firms into the utility function yields the following indirect utility function:

$$V_j = \theta \left[\frac{\alpha}{2} \Delta_j^o + (1 - \alpha) \Delta_j^c \right] \quad (32)$$

An additional variety produced by superstars has a lower impact on V_j than an additional variety

produced by the fringe. To see this, let us assume that that consumers equally like the goods produced by oligopolist and competitive firms, i.e. $\alpha = 0.5$. An additional variety from superstars yields an increase in utility which is half of that arising from an additional variety from the competitive fringe. The result is due to the incomplete pass-through of superstars. In fact, only half of a reduction in costs is passed to consumers through lower prices, while the pass-through is complete for the competitive fringe.

Taking log derivative of the indirect utility function yields:

$$d \ln V_j = \frac{\theta}{V_j} \left[\frac{\alpha}{2} \Delta_j^o d \ln \Delta_j^o + (1 - \alpha) \Delta_j^c d \ln \Delta_j^c \right] \quad (33)$$

Using the envelope theorem

$$d \ln V_j = \lambda_j \frac{y_j}{V_j} d \ln W_j \quad (34)$$

where the marginal utility of income equals:

$$\lambda_j = \frac{\theta}{y_j} \left[\frac{\alpha}{\theta + 2} \Delta_j^o + \frac{(1 - \alpha)}{\theta + 1} \Delta_j^c \right] \quad (35)$$

Substituting yields:

$$\begin{aligned} d \ln W_j &= \frac{\theta}{\lambda_j y_j} \left[\frac{\alpha}{2} \Delta_j^o d \ln \Delta_j^o + (1 - \alpha) \Delta_j^c d \ln \Delta_j^c \right] \\ &= \left[\frac{\theta + 2}{2} \frac{\frac{\alpha}{\theta + 2} \Delta_j^o}{\frac{\alpha}{\theta + 2} \Delta_j^o + \frac{1 - \alpha}{\theta + 1} \Delta_j^c} d \ln \Delta_j^o + (\theta + 1) \frac{\frac{1 - \alpha}{\theta + 1} \Delta_j^c}{\frac{\alpha}{\theta + 2} \Delta_j^o + \frac{1 - \alpha}{\theta + 1} \Delta_j^c} d \ln \Delta_j^c \right] = \\ &= \left[\frac{\theta + 2}{2} \mu_j^o d \ln \Delta_j^o + (\theta + 1) \mu_j^c d \ln \Delta_j^c \right] \end{aligned}$$

The welfare change is a weighted average of the change in the mass of varieties of superstars and fringe available for consumption. The weights depend on the expenditure shares of the two types of goods, and on θ . Using (30), we can write the change in Δ_j^o and Δ_j^c as:

$$d \ln \Delta_j^o = d \ln \delta_{jj}^o + d \ln \mu_j^o - d \ln s_{jj}^o \quad d \ln \Delta_j^c = d \ln \delta_{jj}^c + d \ln \mu_j^c - d \ln \Lambda_{jj}^c$$

Using the definition of the optimal scope (28) yields:

$$\begin{aligned} d \ln \Delta_j^o &= d \ln \mu_j^o + \frac{\theta}{\theta + 1} \left[1 + \frac{s_{jj}^o}{\theta(1 - s_{jj}^o)} \right] (-d \ln s_{jj}^o) \\ d \ln \Delta_j^c &= d \ln \mu_j^c + \frac{\theta}{\theta + 1} (-d \ln \Lambda_{jj}^c) \end{aligned}$$

The aggregate mass of varieties produced by a group of firms positively depends on their expenditure share. Moreover, Δ_j^o falls with s_{jj}^o and Δ_j^c falls with Λ_{jj}^c . Using these results in the welfare

formula, and given that $d \ln \mu_j^o = -\frac{\mu_j^c}{\mu_j^o} d \ln \mu_j^c$, yields the welfare formula for the welfare gains from a small reduction in τ in such a model:

$$d \ln W_j = \underbrace{\mu_j^o \left[\frac{\theta(\theta + 2)}{2(\theta + 1)} \right] \left[1 + \frac{s_{jj}}{\theta(1 - s_{jj})} \right]}_{\text{Large Multiproduct Firms}} (-d \ln s_{jj}) + \underbrace{\mu_j^c \theta (-d \ln \Lambda_{jj}^c)}_{\text{Competitive Fringe}} + \underbrace{\frac{\theta}{2} \mu_j^c d \ln \mu_j^c}_{\text{Interaction}} \quad (36)$$

In the main text of the paper I claimed that the contribution of the last two terms is overall positive. That is, the interaction term cannot offset the competitive fringe term. To see this, let us rearrange the equation as:

$$d \ln W_j = \mu_j^o \left[\frac{\theta(\theta + 2)}{2(\theta + 1)} \right] \left[1 + \frac{s_{jj}}{\theta(1 - s_{jj})} \right] (-d \ln s_{jj}) + \mu_j^c \frac{\theta}{2} (-d \ln \Lambda_{jj}^c) + \frac{\theta}{2} (-d \ln \tilde{\Lambda}_{jj}^c) \quad (37)$$

where

$$\tilde{\Lambda}_{jj}^c = \frac{1}{1 + \left(\frac{w_j c_j^c}{w_i c_i^c \tau_{ij}} \right)^{\frac{1}{\theta}}} \quad (38)$$

which is increasing in τ_{ij} .

1.7 Welfare Gains in Monopolistic Competition

This section studies the welfare gains from trade in a model of multiproduct firms that are monopolistically competitive. The model is identical to the baseline case except for the market structure assumption of monopolistic competition. Firms are infinitesimally small and do not face cannibalization effects by construction. In fact, when firms maximize their profits they take the marginal utility of income λ_j as given. In such an environment, the optimal scope of a firm is implicitly defined by:

$$c_{kij}(\delta_{kij}) = \frac{1}{\lambda_j \bar{q}} \quad (39)$$

Cannibalization effects are absent in this case. The optimal quantity supplied by a firm is identical to the baseline model:

$$x_{kij}(\omega) = \bar{q} L_j \left[\left(\frac{c_{kij}(\delta_{kij})}{c_{kij}(\omega)} \right)^{\frac{1}{2}} - 1 \right] \quad (40)$$

On the other hand, prices are not a function of the market share of the firm as in the baseline model.

$$p_{kij}(\omega) = [c_{kij}(\omega) c_{kij}(\delta_{kij})]^{\frac{1}{2}} \quad (41)$$

With our functional form for the marginal cost of production and delivery, we can fully characterize the equilibrium in closed form. The scope of firms is given by:

$$\delta_{kij} = \left[\frac{\theta + 2}{\theta \bar{q} w_i c_{ki} \tau_{ij}} \right]^{\frac{1}{\theta+1}} y_j^{\frac{1}{\theta+1}} [s_{kij}]^{\frac{1}{\theta+1}} \quad (42)$$

Profits equal:

$$\Pi_i = \frac{\theta s_{ii}}{\theta + 1} y_i L_i + \frac{\theta s_{ij}}{\theta + 1} y_j L_j - w_i F$$

Using market clearing and the zero profit condition, the number of firms equals:

$$M_i = \frac{L_i \theta}{F(\theta + 1)} \quad (43)$$

I focus on the equilibrium with symmetric firms. Since the optimal quantity per firm is the same as in the baseline case, the indirect utility function equals:

$$V_j = \frac{\theta}{2} \Delta_j \quad (44)$$

The marginal utility of income has an identical expression to the baseline case. Hence, the only difference between the baseline model and a model of multiproduct firms' that are monopolistically competitive lies in the equilibrium value of the mass of varieties available for consumption Δ_j :

$$\Delta_j^{\theta+1} = s_{jj}^{-\theta} \frac{(\theta + 2)}{\bar{q} \theta c_j}$$

Differentiating and following the steps shown in the paper, we obtain the welfare formula for the change in welfare following a small reduction in τ :

$$d \ln W_j^{MC} = \frac{\theta(\theta + 2)}{2(\theta + 1)} (-d \ln s_{jj})$$

1.8 Social Planner's Allocation

In this section I compare the social planner's allocation to that emerging from a model of monopolistic competition and oligopoly. The goal of this section is to illustrate the sources of distortions present in monopolistic competition and oligopoly, to shed light on the sources of the welfare gains from trade and on the difference between non-homothetic preferences and homothetic preferences.

I focus on a closed economy for simplicity. The main result of this section also hold for a two-country case as well.

1.8.1 Non-Homothetic Preferences - Planner's Allocation

The social planner chooses the quantity of each variety ω , the number of firms M and the scope of each firm δ to maximize consumer's utility:

$$U = M \int_0^\delta [\ln(q(\omega) + \bar{q}) - \ln \bar{q}] d\omega$$

subject to the resource constraint:

$$L \geq M \left[\int_0^\delta Lc(\omega)q(\omega)d\omega \right]$$

Let κ denote the Lagrangian multiplier associated with the resource constraint. The first order conditions of the planner's problem are:

$$\begin{aligned} \{q(\omega)\} : \quad & \frac{1}{q(\omega) + \bar{q}} = \kappa c(\omega)L \\ \{\delta\} : \quad & \ln(q(\delta) + \bar{q}) - \ln(\bar{q}) = \kappa c(\delta)q(\delta)L \\ \{M\} : \quad & \int_0^\delta [\ln(q(\omega) + \bar{q}) - \ln \bar{q}] d\omega = \kappa \left[\int_0^\delta Lc(\omega)q(\omega)d\omega \right] \\ \{\kappa\} : \quad & L = M \left[\int_0^\delta Lc(\omega)q(\omega)d\omega \right] \end{aligned}$$

I use the functional form assumption for the marginal cost of production: $c(\omega) = c\omega^\theta$. Moreover, to easily compare the scope of firms across allocation, I let $s = \frac{1}{M}$ denote the market share of firms in the planner's allocation. The optimal supply, scope and number of firms is given by:

$$\begin{aligned} q(\omega)^P &= \bar{q} \left[\left(\frac{\delta^P}{\omega} \right)^\theta - 1 \right] \\ \delta^P &= \left[\frac{1}{\theta \bar{q} c} \right]^{\frac{1}{\theta+1}} (s^P)^{\frac{1}{\theta+1}} \\ s^P &= \frac{(\theta + 1)F}{L\theta} \end{aligned}$$

1.8.2 Non-Homothetic Preferences - Market VS Planner

Let us compare the planner's allocation to the market allocation generated by my baseline model, and a model of monopolistic competition in a closed economy. Let superscript MC and O denote

the allocation in monopolistic competition and oligopoly.

$$\begin{aligned}
q(\omega)^{MC} &= \bar{q} \left[\left(\frac{\delta^{MC}}{\omega} \right)^{\frac{\theta}{2}} - 1 \right] \\
\delta^{MC} &= \left[\frac{\theta + 2}{\theta \bar{q} c} \right]^{\frac{1}{\theta+1}} (s^{MC})^{\frac{1}{\theta+1}} \\
s^{MC} &= \frac{(\theta + 1)F}{L\theta}
\end{aligned}$$

First, the MC allocation generates the socially optimal level of market concentration, as the market shares $s^{MC} = s^P$. Second, the MC scope is larger than the socially optimal one, by a constant factor: $\delta^{MC} = (\theta + 2)^{\frac{1}{\theta+1}} \delta^P$. Finally, the ratio $q(\omega)^{MC}/q(\omega)^P$ is increasing in ω and in δ^{MC} .

The market allocation under monopolistic competition is inefficient. Although the market concentration, that is the number of firms, is the same as what a planner would choose, the presence of variable markups within a firm generates a distortion. In particular, high markup varieties are underconsumed and low markup varieties are overconsumed relative to the planner's allocation. As a result, firms produce too many varieties in the market allocation relative to the social planner. Such distortion is named *business stealing bias* by [Dhingra and Morrow \(2016\)](#). Monopolistically competitive firms do not internalize the business stealing produced by new varieties: they are able to sell additional varieties far from the core because they charge low markups for those varieties.

How do distortions change in the oligopoly allocation? The allocation in the oligopoly model is defined by:

$$\begin{aligned}
q(\omega)^O &= \bar{q} \left[\left(\frac{\delta^O}{\omega} \right)^{\frac{\theta}{2}} - 1 \right] \\
\delta^O &= \left[\frac{\theta + 2}{\theta \bar{q} c} \right]^{\frac{1}{\theta+1}} (s^O(1 - s^O))^{\frac{1}{\theta+1}} \\
(s^O)^2 + \theta s^O &= \frac{(\theta + 1)F}{L}
\end{aligned}$$

The oligopoly allocation yields lower welfare than both the planners allocation and the MC allocation. However, oligopoly affects welfare through the scope, pricing and entry decisions of firms, and each of this variables affect welfare differently.

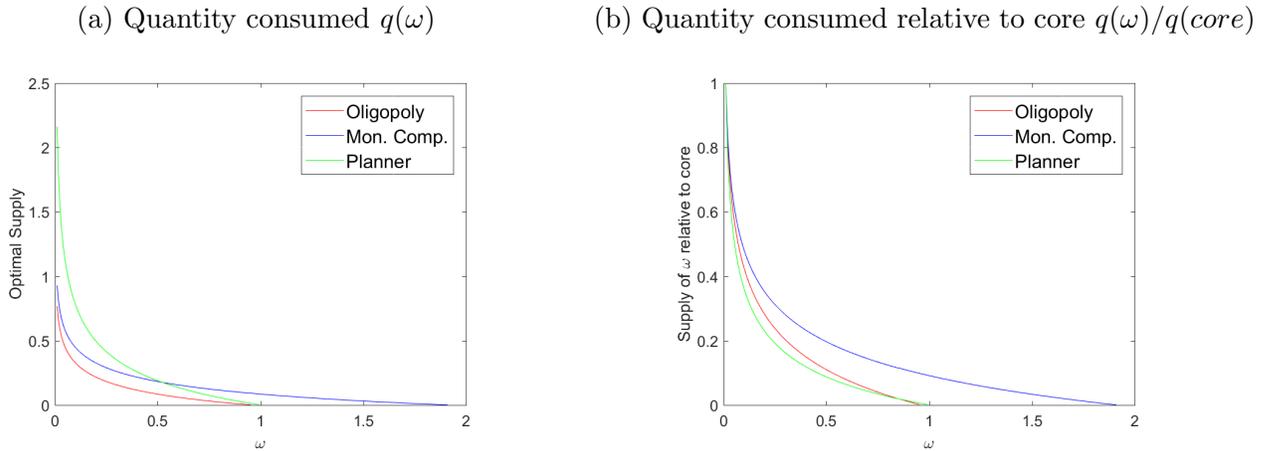
Let us first consider the scope decisions of firms. The oligopoly allocation features the same distortion of the MC allocation: high-markup varieties are under consumed, and low-markup varieties are overconsumed. However, the presence of cannibalization effects on scope partially reduces such a distortion. The distortion in the MC allocation arises because firms do not internalize the business stealing effects of new low-markup varieties. Oligopolist firms, on the other hand, partially internalize the business stealing bias: firms internalize the within-firm business stealing, which is

what cannibalization effects are. Holding constant the market share, the scope of oligopolistic firms is lower than the MC scope, and, depending on s^O , it is larger, equal or smaller than the planner's scope.

Second, cannibalization has a direct effect on markups. By reducing the scope due to cannibalization, firms are restricting their supply and, thus, able to charge higher markups across all their sold varieties. The effect of cannibalization on markups causes an under-consumption of all varieties, which reduces welfare relative to the MC allocation. In welfare terms, the negative effects of cannibalization on welfare through markups dominates the positive welfare effects of cannibalization on scope.

To gain a better understanding of how monopolistic competition, and cannibalization effects on scope and prices affect welfare, consider figure 4. I hold constant the market share across the three allocations, set $s^O = 0.5$ and normalize δ^P to one. Panel 4a plots the quantity consumed $q(\omega)$ for each variety under the three allocation. Comparing the planner's allocation to the MC one highlights the overconsumption of low markup varieties, and the under consumption of high markup varieties. Because of cannibalization effects on prices, all varieties are under consumed in the oligopoly allocation. Panel 4b shows the relative consumption of a variety ω relative to the core¹⁰. The oligopoly allocation, depending on the level of market share, approaches the relative consumption of varieties chosen by a planner.

Figure 4: Monopolistic Competition, Oligopoly and Social Planner



Finally, given the same fundamental parameters of the model (L, F) , oligopoly generates firms that have lower market shares relative to the planner's allocation. Since oligopolists can extract larger profits from firms, the oligopoly allocation features more firms than the planner's and MC allocation. As the scale of each firm is smaller, the overall effect is an additional under-consumption of varieties.

¹⁰As the core variety has infinite consumption in the model, I consider a variety with $\omega = 0.01$.

1.8.3 Comparison to CES preferences

Under CES preferences, the monopolistically competitive allocation is identical to the planner's allocation (Melitz and Redding, 2015). The oligopoly allocation is, however, inefficient. Since markups within a firm are constant, the inefficiency does not depend on over or under consumption of varieties within a firm. Cannibalization effects on scope cause too few varieties to be produced. Cannibalization effects on prices cause all varieties to be underconsumed. Larger entry in oligopoly, relative to MC causes an additional underconsumption of varieties¹¹.

A reduction in trade costs causes a fall in the market share of domestic producers. The fall in market share causes the weakening of cannibalization effects on prices and scope. While cannibalization effects on prices and scope are both sources of distortions in the CES model, cannibalization effects on scope partially reduces the business stealing bias in the non-homothetic model. As a result, a reduction in trade costs generates larger gains in the CES model relative to my baseline model.

1.9 Single Product Firm Model

In this section, I derive the formula for the welfare gains from trade in the presence of large single product exporters, in a model where consumers have CES preferences of the following form:

$$U_j = \left[\sum_{i=h,f} \sum_{k=1}^{M_i} q_{kij}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution across goods. All goods enter the function symmetrically: given the same quantity, the utility value of a new variety is independent of the firm producing it. As in the baseline model M_i is the number of firms from i . Solving the consumer's problem yields the following aggregate inverse demand function for the variety produced by firm k from i to j :

$$p_{kij} = \left[\frac{y_j L_j}{\sum_{i=h,f} \sum_{k=1}^{M_i} x_{kij}^{\frac{\sigma-1}{\sigma}}} \right] x_{kij}^{-\frac{1}{\sigma}} = A_j x_{kij}^{-\frac{1}{\sigma}}$$

where $x_{kij}(\omega) = L_j q_{kij}(\omega)$. To derive the welfare formula, it is convenient to write the demand shifter A_j as a function of U_j .

$$A_j = \frac{y_j L_j^{\frac{1}{\sigma}}}{U_j^{\frac{\sigma}{\sigma-1}}} \quad (45)$$

Moreover, the marginal utility of income equals:

$$\lambda_j = \frac{U_j}{y_j} \quad (46)$$

¹¹Details on derivations are available upon request.

Each firm produces a differentiated good, pays a fixed cost, independent of quantity, and competes à la Cournot. The marginal cost of production and delivery is $c_{kij} = w_i \tau_{ij} c_{k,i}$. The optimal pricing equation is:

$$p_{kij} = \frac{\sigma}{(\sigma - 1)(1 - s_{kij})} w_i \tau_{ij} c_{k,i}$$

As in the baseline model, I assume that firms from the same country of origin are symmetric, and that free entry drives profits to zero. Without loss of generality, I normalize the per capita income in country j to 1. To obtain the welfare formula, let us use the definition of the domestic market share of a firm from j :

$$r_{jj} = \left(\frac{(\sigma - 1)(1 - s_{jj})}{\sigma c_j} \right)^{\sigma - 1} A_j^\sigma = s_{jj} L_j$$

Using (45), I can re-write the indirect utility function in j as:

$$V_j = \left(\frac{\sigma - 1}{\sigma c_j} \right)^{\sigma - 1} s_{jj}^{-1} (1 - s_{jj})^{\sigma - 1}$$

Given (46), $d \ln V_j = d \ln W_j$. Hence, our welfare formula becomes:

$$d \ln W_j = \left[\frac{1}{\sigma - 1} + \frac{s_{jj}}{1 - s_{jj}} \right] (-d \ln s_{jj}) \quad (47)$$

The gravity equation that arises in this model is:

$$\Lambda_{ij} = \frac{M_i \left(\frac{1 - s_{ij}}{\tau_{ij} w_i c_i} \right)^{\sigma - 1}}{\sum_v M_v \left(\frac{1 - s_{vj}}{\tau_{vj} w_v c_v} \right)^{\sigma - 1}}$$

In a model with small firms, the market share of each firm converges to zero, and the mass of entrants M_i is a constant. Hence, in a model of monopolistic competition, the trade elasticity is $\epsilon = \sigma - 1$ as in the model of [Krugman \(1980\)](#). For $s_{jj} = 0$, (47) is the same as the one derived by ACR for the Krugman model.

1.10 How large is the mismeasurement

The paper showed the welfare change from 2002 to 2007 using as s_{jj} the average market share of the largest 4 firms. Here I report the welfare change using alternative measures of s_{jj} . In table 7, s_{jj} is the average market share of the largest 8 firms. In table 8, s_{jj} is the average market share of the largest 4 firms, and I assume that the export share is 50% of s_{jj} . Finally, in table 9), s_{jj} is the average market share of the largest 4 firms, and I assume that the export share is 1% of s_{jj} .

Table 7: Welfare Gains Across Models (2007-2002 - 8 Largest Firms)

	$d \ln W(\%)$	% Diff
Baseline	3.2 (4.2)	—
Stone-Geary, Monop. Comp	2.8 (3.8)	-13.1 (8.1)
Stone-Geary, Constant MUI	4.7 (9.9)	38.0 (34.5)
CES, Oligopoly	4.9 (5.7)	74.1 (52.9)
CES, Monop. Comp	3.4 (5.1)	0.4 (15.1)

The table reports $d \ln W$ and the % Difference ($W_m/W_{Baseline} - 1$) relative to the baseline model averaged across industries. Standard errors in parenthesis. The averages are weighted by the industry absorption. All values are in percentages.

Table 8: Welfare Gains Across Models (2007-2002 - 4 Largest Firms - $s^* = 0.5s$)

	$d \ln W(\%)$	% Diff
Baseline	4.2 (6.1)	—
Stone-Geary, Monop. Comp	3.8 (5.6)	-11.0 (7.0)
Stone-Geary, Constant MUI	6.0 (13.1)	33.6 (33.8)
CES, Oligopoly	6.5 (8.5)	85.4 (64.7)
CES, Monop. Comp	4.5 (7.4)	2.8 (13.5)

The table reports $d \ln W$ and the % Difference ($W_m/W_{Baseline} - 1$) relative to the baseline model averaged across industries. Standard errors in parenthesis. The averages are weighted by the industry absorption. All values are in percentages.

Table 9: Welfare Gains Across Models (2007-2002 - 4 Largest Firms - $s^* = 0.01s$)

	$d \ln W(\%)$	% Diff
Baseline	4.7 (6.7)	—
Stone-Geary, Monop. Comp	3.8 (5.6)	-20.7 (11.4)
Stone-Geary, Constant MUI	7.1 (14.2)	38.7 (34.6)
CES, Oligopoly	6.7 (8.8)	65.8 (46.3)
CES, Monop. Comp	4.5 (7.4)	-8.1 (17.9)

The table reports $d \ln W$ and the % Difference ($W_m/W_{Baseline} - 1$) relative to the baseline model averaged across industries. Standard errors in parenthesis. The averages are weighted by the industry absorption. All values are in percentages.

Table 10: Parameters

Industry	σ	ϵ
Auto	3.50	1.84
Basic metals	4.28	3.28
Chemicals	3.07	3.13
Communication	2.75	3.95
Electrical	2.23	12.91
Food	4.45	2.62
Machinery n.e.c.	5.45	1.45
Medical	2.87	8.71
Metal products	2.41	6.99
Minerals	2.48	2.41
Office	4.68	12.95
Other	2.46	3.98
Other Transport	5.97	0.39
Paper	2.80	16.52
Plastic	3.25	1.67
Textile	4.85	8.1
Wood	3.87	11.5

Table 11: θ across models. Baseline sample.

Code	Industry	θ^{MC}	$\theta^{Baseline}$	θ^{CES}
3111	Animal food manufacturing	0.38	0.37	0.54
3112	Grain and oilseed milling	0.38	0.36	0.54
3113	Sugar and confectionery product manufacturing	0.38	0.37	0.54
3114	Fruit and vegetable preserving and specialty food manufacturing	0.38	0.37	0.54
3115	Dairy product manufacturing	0.38	0.37	0.54
3116	Animal slaughtering and processing	0.38	0.37	0.54
3117	Seafood product preparation and packaging	0.38	0.37	0.54
3118	Bakeries and tortilla manufacturing	0.38	0.37	0.54
3119	Other food manufacturing	0.38	0.37	0.54
3121	Beverage manufacturing	0.38	0.37	0.54
3122	Tobacco manufacturing	0.38	0.35	0.53
3141	Textile furnishings mills	0.12	0.11	0.31
3149	Other textile product mills	0.12	0.12	0.31
3222	Converted paper product manufacturing	0.06	0.05	0.19
3231	Printing and related support activities	0.06	0.06	0.20
3254	Pharmaceutical and medicine manufacturing	0.32	0.31	0.46
3255	Paint, coating, and adhesive manufacturing	0.32	0.31	0.46
3256	Soap, cleaning compound, and toilet preparation manufacturing	0.32	0.30	0.46
3259	Other chemical product and preparation manufacturing	0.32	0.31	0.46
3261	Plastics product manufacturing	0.60	0.60	0.64
3262	Rubber product manufacturing	0.60	0.59	0.64
3271	Clay product and refractory manufacturing	0.41	0.41	0.50
3272	Glass and glass product manufacturing	0.41	0.41	0.50
3322	Cutlery and handtool manufacturing	0.14	0.14	0.29
3341	Computer and peripheral equipment manufacturing	0.08	0.07	0.24
3342	Communications equipment manufacturing	0.25	0.24	0.40
3346	Manufacturing and reproducing magnetic and optical media	0.08	0.07	0.20
3351	Electric lighting equipment manufacturing	0.08	0.07	0.20
3352	Household appliance manufacturing	0.08	0.06	0.20
3361	Motor vehicle manufacturing	0.54	0.52	0.62
3366	Ship and boat building	2.56	2.54	1.45
3369	Other transportation equipment manufacturing	2.56	2.55	1.46
3371	Household and institutional furniture and kitchen cabinet manufacturing	0.09	0.08	0.25
3399	Other miscellaneous manufacturing	0.25	0.25	0.39

2 Empirical Appendix

2.1 Data Description

The Exporter Dynamics Database is a new dataset from the World Bank that reports transaction-level customs data (Fernandes et al., 2016). The dataset covers eleven source countries: Albania, Burkina Faso, Bulgaria, Guatemala, Jordan, Malawi, Mexico, Peru, Senegal, Uruguay, and Yemen from 1993 to 2011. The sources for the data for each country are detailed in the Annex of Cebeci et al. (2012). The data was collected by the Trade and Integration Unit of the World Bank Research Department, as part of their efforts to build the Exporter Dynamics Database.

Table 12: Descriptive Statistics over the Sample of Countries of Origin

Country	Years	Share of MPF (number, %)	Share of MPF (value, %)	Share of Top 5% (value, %)	Share of Top 1% (value, %)
Albania	2004-2012	50.5	80.5	37.3	14.0
Bulgaria	2001-2006	60.8	91.1	49.8	21.9
Guatemala	2003-2010	63.4	87.9	56.5	25.4
Jordan	2003-2012	37.2	81.3	50.0	17.9
Mexico	2000-2006	48.1	83.5	62.5	39.7
Peru	1993-2009	62.4	81.0	57.9	28.6
Senegal	2000-2012	46.3	77.3	38.9	12.9
Uruguay	2001-2012	46.4	90.8	61.5	18.6
Yemen	2008-2012	53.6	89.9	42.9	9.3
Average	-	52.1	84.8	50.8	20.9

Year availability by country. The second column reports the ratio between the number of multiproduct firms (MPF) and the total number of exporters per country, averaged across years. The third column reports the share of export value of MPF. The last two columns show the share of export value of the top 5% and 1% of MPF by sales. Sample: consumption goods.

I drop firms and products that are not classified (“OTH”), and duplicates. Following Freund and Pierola (2015), I drop firms with less than \$1000 worth of export, and drop Chapter 27 according to the HS classification: mineral fuels, oils and product of their distillation; etc. I restrict the sample to consumption goods by matching each HS6 digits good with the corresponding BEC category, and keeping only the BEC categories that, according to UN Comtrade, correspond to consumption goods: 112, 122, 522, 61, 62 and 63.

I define multiproduct firms as those firms that sell at least two products in one destination in a year. Since I divide multiproduct firms into percentiles, I drop all origin-year pairs in which the number of multiproduct firm is less than 100. This procedure automatically drops Malawi and Burkina Faso. Moreover, I drop all Mexican observations after 2006. In 2007, in fact, there is a change in the identification scheme used for firms, which seats uneasy with the empirical model. Moreover, in 2007, each firms is reported twice, once with the old identifier, once with the new one.

Table 12 demonstrates that multiproduct firms, and especially large multiproduct firms, dominate world trade. While about half of the firm in each country export more than one variety, more than 80% of the total export of a country are made by multiproduct firms. The top 1% of exporters accounts, on average, for 20% of the country's export of consumption goods, with the highest value reached by Mexican firms (40%). On average, the top 5% of exporters accounts for 50% of a country total exports.

Are Mexican firms large in their destination markets? To answer this question, I consider the industry specific market shares of Mexican exporters, defined as total firm exports over total industry imports of the destination (SITC 2 digit). The top 1% of Mexican exporters is large in Central and South America: the average market share of the top 1% is 9.6% in Belize, 4.8% in Nicaragua and 2% in Peru and Colombia, while it is 0.5% in the US. Moreover, the largest market share attained by Mexican firms in the destination is 28% in Belize, 23.3% in Nicaragua, 20.4% in Colombia, 17.7% in Peru, and 5.6% in USA.

An online search of the largest Mexican exporters of consumers goods highlighted, among others, three large producers. FEMSA (250k employees, \$23bn in revenues), sells beverages in South America - together with Coca-Cola. Grupo Bimbo (129k employees, \$14bn in revenues) is the largest bakery producer in the world, as it acquired several brands in recent years. Finally, Grupo Modelo (40k employees, \$7bn revenues) is a brewery, famous for the production of Corona.

Let us take a closer look at the industry composition of Mexican multiproduct exporters. Table 13 illustrates the share of firms in the bottom 95%, top 5% and 1% of the distribution in each industry. The bottom 95% of firms by total sales is dominated by firms producing textile, miscellaneous and vegetable products. The machinery and electrical industry, and textiles are over represented in the top percentiles of the distribution. Firms in the machinery and electrical industry account for 37% and 13.3% of the top 1% and 5% of firms, while only 3.4% of the bottom 95% of firms belongs to such an industry. 20% of firms in the bottom 95% export textile goods, while that percentage increases to 31.2% in the top 1%. The production of vegetable products and footwear is underrepresented in the highest percentiles of the distribution. The industry composition of Mexican exporters to the US is similar to the general one.

How does market concentration vary across industries? Table 14 reports the average market shares of firms in each bin and industry. The market share is defined as export of a firm in a destination over the total industry Mexican export to the same destination. The industry of a multi-industry firm is its top selling industry. Consider the average market share of firms selling to the US (the results averaged across destination are similar). The skins & leather, metals, and transportation industries are the most concentrated, where the top 1% of Mexican exporters controls 45%, 27.1% and 55.3% of the industry export.

Table 13: Industry Composition by Bin of Mexican Multiproduct Firms

HS	Industry	All Destinations			To the US		
		Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
01-05	Animals Prod.	4.0	1.6	1.2	4.3	1.6	1.3
06-15	Vegetable Prod.	15.4	13.7	1.8	17.2	13.9	1.8
16-24	Foodstuffs	6.5	9.8	9.8	6.2	9.8	9.6
28-38	Chemicals	4.4	8.3	5.0	2.9	7.0	4.7
39-40	Plastic & Rubber	5.4	3.5	1.7	5.2	3.6	1.7
41-43	Skins & Leather	2.8	0.8	1.2	2.7	0.8	1.3
44-49	Wood Prod.	4.2	2.9	1.6	3.7	2.9	1.6
50-63	Textiles	20.5	38.1	31.5	21.0	38.8	31.7
64-67	Footwear & Headgear	7.1	1.2	-	7.4	1.2	-
68-71	Stone & Glass	6.5	2.1	1.8	6.4	2.1	1.8
72-83	Metals	3.4	0.6	1.6	3.3	0.6	1.7
84-85	Machinery & Electrical	3.4	7.2	36.9	3.1	7.3	37.1
86-89	Transportation	0.3	0.5	1.2	0.2	0.5	1.3
90-97	Miscellaneous	16.2	9.8	8.7	16.4	10.0	8.7

Let $M_{M,i,b,t}$ be the number of multiproduct firms from Mexico in bin b in industry i at year t . The bins b are listed in the columns, and the industries are the rows of the table. The columns grouped under “All Destinations” report $T^{-1} \sum_{t=1}^T (M_{M,i,b,t} / \sum_i M_{M,i,b,t})$, which is the share of firms of bin b in industry i , averaged across years. The columns grouped under “To the US” report the same statistics for firms exporting to the US.

Table 14: Average Market Shares of Mexican Multiproduct Firms

HS	Industry	All Destinations			To the US		
		Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
01-05	Animals Prod.	0.9	4.3	9.8	0.1	1.8	9.5
06-15	Vegetable Prod.	0.2	0.8	7.6	0.0	0.7	7.5
16-24	Foodstuffs	0.4	1.6	6.7	0.1	0.8	5.5
28-38	Chemicals	0.7	2.5	14.1	0.1	1.9	10.6
39-40	Plastic & Rubber	0.6	3.5	9.3	0.1	3.5	9.3
41-43	Skins & Leather	1.8	13.5	45.0	0.3	13.3	45.0
44-49	Wood Prod.	0.7	5.3	18.7	0.1	4.4	18.7
50-63	Textiles	0.4	0.5	1.3	0.0	0.4	1.3
64-67	Footwear & Headgear	0.8	6.9	-	0.1	6.9	-
68-71	Stone & Glass	0.7	7.1	16.8	0.1	6.6	16.8
72-83	Metals	1.3	16.3	42.9	0.2	16.1	27.1
84-85	Machinery & Electrical	0.4	0.4	2.9	0.0	0.3	2.8
86-89	Transportation	13.2	31.2	55.3	0.7	31.2	55.3
90-97	Miscellaneous	0.3	1.1	5.8	0.0	1.1	5.6

The table reports the average market share of firms by bin and industry. In the three columns under “All Destinations”, the firm’s market share is defined as the total export of the firm across all destinations, over firm’s industry total exports. The columns grouped under “To the US” report the same statistics for firms exporting to the US. The measure of market share is averaged across years.

2.2 Robustness Checks for the First Stylized Fact

The first stylized fact documented by the paper is that export superstars tend to export more varieties each in richer economies. Recall that our baseline regression is:

$$\ln(\# \text{ Products}_{kMjt}) = \beta_0 + \beta_y \ln(\text{Pc. Income}_{jt}) + \beta_L \ln(\text{GDP}_{jt}) + \beta_\tau \tau_{Mjt} + f_k + g_t + \epsilon_{kMjt} \quad (48)$$

Using Mexican data, the paper documented a positive and statistically significant coefficient β_y for the largest multiproduct firms. For the bottom 95% of exporters, the coefficient was economically small and insignificant. This section shows that the finding is robust to a number of alternative model specifications.

I start, in Section 2.2.1, by repeating the analysis using firm-year fixed effects and focusing on 2004. Using firm-year fixed effect has no impact on the findings on the paper. Results are robust when we consider 2004: only the top 1% of exporters sells more varieties to richer economies controlling for size.

Second, I consider the whole sample of source countries available in the Exporter Dynamics Database. In Section 2.2.2, I show the results from the baseline regression of the model using 1) the entire sample of countries (with origin-year fixed effects), and 2) only Peru as an example. In both cases, the coefficient on per capita income is positive and significant for the top 5% and 1% of exporters. The elasticity of the scope with respect to per capita income is lower than the one found using only Mexican data.

The third set of robustness checks involves alternative set of geographical controls and definitions of distance (Section 2.2.3). First, I extend the set of controls to include dummies for regional trade agreements, commonality of legislation, and a dummy that equals 1 if the destination is a member of WTO. Moreover, I consider the distance dummies of Eaton and Kortum (2002), and a third-degree polynomial for $\log(\text{distance})$. I report the results by group: table 17 shows the result for the top 1% of exporters, 18 for the top 5%, and 19 for the bottom 95%. The estimated β_y is robust to those changes.

Finally, I consider different definitions of per capita income. I estimate (48) using different measures of per capita income: nominal per capita GDP, PPP-adjusted per capita GDP, GNI measured according to the Atlas method, GNI, and household consumption finding similar results. I report the results by group: table 20 shows the result for the top 1% of exporters, 21 for the top 5%, and 22 for the bottom 95%. Across the different measures, β_y is in line with the finding of the baseline regression. The only outlier is the coefficient on PPP-adjusted per capita GDP, which is almost twice as the baseline one.

A possible issue that biases the results is the selection of firms into the three groups considered, because of the correlation between sales and total number of products that a firm exports. Such a correlation is however small (Figure 5), as there are small and wide scope firms at any level of the distribution of firms' sales. A similar pattern appears in the distribution of destinations reached by

firms. Although the largest number of destinations is reached by the largest firms, there are firms that reach 1 to 7 destinations at any level of the distribution. To mitigate the potential selection, I consider two alternative ways to divide firms into groups. First, I divide firms in bins by the lagged total sales (First three columns of table 23). Second, I use the distribution of multiproduct firms' sales in the US, the most popular destination for Mexico (Last three columns of table 23). Results are robust to both alternatives. Another potential source of selection bias is the change in the industry composition of our groups of firms. In other words, although we may be thinking of selecting the largest firms, we may very well select the firms of a particular industry, whose firms tend to have large revenues. An alternative solution is to divide firms in percentiles within each industry. Results are robust to this specification (Last three columns of Table 23).

The analysis has so far focused on the bottom 95% and the top 5% and 1% of multiproduct firms. The last set of robustness checks (section 2.2.5), considers alternative distributions of firms. First, I consider all firms (not only multiproduct exporters). For each year, I divide the firms in percentile by sales and repeat the empirical analysis with such a distribution. Table 24 shows that the main result of the paper are robust to this alternative division of firms in bins. [Bernard et al. \(2015\)](#) found that the partial year bias could have relevant consequences in empirical trade studies. When firms enter a market they are registered as exporters for that year regardless of the number of months the firm has operated. As a result, a firm that starts exporting in December may be 1/12 smaller than an identical firm that started selling in January. A similar problem would arise if the firm exit the market. To avoid such a problem, I drop new entrants and firms that exit from my sample (thus eliminating year 2000 and 2006). Results (table 24) are robust. Furthermore, I test the first hypothesis of the model using deciles of the distribution of firms. Details available upon request.

2.2.1 Robustness Checks

Table 15: Per Capita Income and Product Scope of Multiproduct Exporters

	Baseline			Firm year FE			2004		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
Log(Pc.income)	0.028 (0.023)	0.065*** (0.023)	0.113*** (0.037)	0.027 (0.022)	0.067*** (0.023)	0.116*** (0.037)	0.026 (0.026)	0.023 (0.023)	0.086** (0.039)
logGDP	0.055*** (0.014)	0.102*** (0.015)	0.161*** (0.023)	0.055*** (0.012)	0.103*** (0.015)	0.163*** (0.023)	0.046*** (0.014)	0.118*** (0.018)	0.188*** (0.030)
Log(Distance)	-0.193*** (0.061)	-0.357*** (0.072)	-0.550*** (0.102)	-0.198*** (0.051)	-0.363*** (0.069)	-0.570*** (0.102)	-0.210*** (0.056)	-0.401*** (0.068)	-0.547*** (0.100)
Border	0.258* (0.148)	0.357** (0.167)	0.265 (0.168)	0.243* (0.136)	0.326* (0.167)	0.232 (0.167)	0.275** (0.123)	0.336** (0.139)	0.352** (0.138)
Comm. Language	0.149** (0.074)	0.329*** (0.087)	0.549*** (0.124)	0.148** (0.062)	0.342*** (0.084)	0.559*** (0.123)	0.177*** (0.064)	0.306*** (0.078)	0.666*** (0.129)
Island	0.025 (0.043)	0.046 (0.062)	0.085 (0.097)	0.017 (0.039)	0.043 (0.061)	0.083 (0.098)	0.017 (0.040)	0.016 (0.075)	0.011 (0.114)
Landlocked	0.015 (0.034)	-0.093 (0.057)	-0.167* (0.101)	0.020 (0.029)	-0.102* (0.059)	-0.176* (0.104)	0.034 (0.035)	-0.036 (0.097)	-0.047 (0.129)
R^2	0.60	0.59	0.67	0.53	0.58	0.68	0.54	0.58	0.70
# Observations	78887	14157	4380	53210	12946	4184	8155	1922	611

Results from OLS of equation (48) for bottom 95% and top 5% and 1% of multiproduct exporters. In the All countries columns I use origin year fixed effects. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Baseline: baseline results. Firm year FE: uses firm year fixed effects. 2004 restrict the sample to 2004.

2.2.2 Alternative Countries of Origin

Table 16: Per Capita Income and Product Scope of Multiproduct Exporters

	All Countries			Peru		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
Log(Pc.income)	0.010 (0.017)	0.033** (0.016)	0.073*** (0.017)	0.039 (0.030)	0.079*** (0.028)	0.097*** (0.035)
logGDP	0.061*** (0.014)	0.100*** (0.023)	0.111*** (0.027)	0.063*** (0.015)	0.111*** (0.026)	0.118*** (0.036)
Log(Distance)	-0.119*** (0.021)	-0.243*** (0.040)	-0.325*** (0.053)	-0.240*** (0.044)	-0.336*** (0.114)	-0.383** (0.162)
Border	0.108*** (0.040)	0.040 (0.062)	-0.026 (0.091)	-0.068 (0.070)	-0.080 (0.133)	-0.152 (0.174)
Comm. Language	0.079** (0.037)	0.121* (0.064)	0.200** (0.083)	0.002 (0.076)	0.028 (0.166)	-0.068 (0.219)
Island	-0.009 (0.039)	-0.053 (0.042)	-0.030 (0.054)	0.004 (0.043)	-0.058 (0.049)	-0.086 (0.062)
Landlocked	0.014 (0.047)	-0.098 (0.078)	-0.124 (0.076)	0.056 (0.098)	0.003 (0.181)	-0.088 (0.179)
R^2	0.57	0.48	0.52	0.59	0.39	0.37
# Observations	236072	47793	13022	56783	15323	4277

Results from OLS of equation (48) for bottom 95% and top 5% and 1% of multiproduct exporters. In the All countries columns I use origin year fixed effects. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%.

2.2.3 Alternative Geographical Controls

Table 17: Per Capita Income and Product Scope of Mexican Multiproduct Exporters

	(Baseline)	(2)	(3)	(4)
Log(Pc.income)	0.113*** (0.037)	0.126*** (0.036)	0.141*** (0.035)	0.123*** (0.040)
Log(GDP)	0.161*** (0.023)	0.163*** (0.022)	0.161*** (0.023)	0.163*** (0.023)
Log(Distance)	-0.550*** (0.102)	-0.589*** (0.105)		40.634** (16.497)
Border	0.265 (0.168)	0.227 (0.182)	0.426** (0.176)	0.295* (0.154)
Comm. Language	0.549*** (0.124)	0.576*** (0.137)	0.564*** (0.133)	0.586*** (0.144)
Island	0.085 (0.097)	0.039 (0.103)	0.085 (0.090)	0.081 (0.089)
Landlocked	-0.167* (0.101)	-0.166 (0.102)	-0.264** (0.131)	-0.099 (0.103)
RTA		-0.132* (0.073)		
GATT/WTO member		0.378*** (0.132)		
Common Leg.		-0.065 (0.100)		
Region 1			1.025*** (0.336)	
Region 2			1.268*** (0.186)	
Region 3			0.633*** (0.131)	
$(Distance)^2$				-4.994** (2.017)
$(Distance)^3$				0.201** (0.082)
R^2	0.67	0.67	0.67	0.67
# Observations	4380	4380	4380	4380

Results from OLS of equation (48) for the Top 1% of Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. The third column uses the Eaton and Kortum (2002) distance regions. A destination is in Region 1 if the bilateral distance with Mexico is below 750 miles, in Region 2 if the distance is between 750 and 1500 miles, and Region 3 if distance is between 1500 and 5000 miles. Region 4 (>5000 miles) is dropped because of collinearity. $(Distance)^x$ is log of distance to the power of x.

Table 18: Per Capita Income and Product Scope of Mexican Multiproduct Exporters

	(Baseline)	(2)	(3)	(4)
Log(Pc.income)	0.065*** (0.023)	0.076*** (0.023)	0.080*** (0.020)	0.067*** (0.024)
Log(GDP)	0.102*** (0.015)	0.103*** (0.015)	0.080*** (0.015)	0.094*** (0.013)
Log(Distance)	-0.357*** (0.072)	-0.381*** (0.073)		42.165*** (9.666)
Border	0.357** (0.167)	0.349** (0.175)	0.672*** (0.101)	0.447*** (0.131)
Comm. Language	0.329*** (0.087)	0.315*** (0.086)	0.384*** (0.077)	0.353*** (0.095)
Island	0.046 (0.062)	0.026 (0.070)	0.053 (0.046)	0.036 (0.052)
Landlocked	-0.093 (0.057)	-0.091 (0.061)	-0.209*** (0.076)	-0.051 (0.057)
RTA		-0.075 (0.051)		
GATT/WTO member		0.203*** (0.069)		
Common Leg.		0.014 (0.063)		
Region 1			0.150 (0.194)	
Region 2			0.718*** (0.107)	
Region 3			0.378*** (0.073)	
$(Distance)^2$				-5.064*** (1.167)
$(Distance)^3$				0.200*** (0.047)
R^2	0.59	0.59	0.59	0.59
# Observations	14157	14157	14157	14157

Results from OLS of equation (48) for the Top 5% of Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. The third column uses the Eaton and Kortum (2002) distance regions. A destination is in Region 1 if the bilateral distance with Mexico is below 750 miles, in Region 2 if the distance is between 750 and 1500 miles, and Region 3 if distance is between 1500 and 5000 miles. Region 4 (>5000 miles) is dropped because of collinearity. $(Distance)^x$ is log of distance to the power of x.

Table 19: Per Capita Income and Product Scope of Mexican Multiproduct Exporters

	(Baseline)	(2)	(3)	(4)
Log(Pc.income)	0.027 (0.023)	0.028 (0.022)	0.020 (0.013)	0.024 (0.018)
Log(GDP)	0.056*** (0.014)	0.059*** (0.014)	0.022** (0.010)	0.043*** (0.010)
Log(Distance)	-0.198*** (0.062)	-0.203*** (0.059)		30.846*** (7.327)
Border	0.259* (0.149)	0.253* (0.152)	0.594*** (0.050)	0.337*** (0.115)
Comm. Language	0.150** (0.074)	0.163** (0.063)	0.212*** (0.044)	0.177** (0.072)
Island	0.023 (0.043)	0.012 (0.054)	0.036 (0.022)	0.024 (0.031)
Landlocked	0.015 (0.033)	0.015 (0.030)	-0.065* (0.033)	0.038 (0.027)
RTA		-0.015 (0.032)		
GATT/WTO member		-0.073* (0.043)		
Common Leg.		-0.011 (0.047)		
Region 1			-0.254*** (0.092)	
Region 2			0.307*** (0.050)	
Region 3			0.093*** (0.034)	
$(Distance)^2$				-3.714*** (0.877)
$(Distance)^3$				0.147*** (0.035)
R^2	0.60	0.60	0.61	0.60
# Observations	80718	80718	80718	80718

Results from OLS of equation (48) for the Bottom 95% of Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. The third column uses the Eaton and Kortum (2002) distance regions. A destination is in Region 1 if the bilateral distance with Mexico is below 750 miles, in Region 2 if the distance is between 750 and 1500 miles, and Region 3 if distance is between 1500 and 5000 miles. Region 4 (>5000 miles) is dropped because of collinearity. $(Distance)^x$ is log of distance to the power of x.

2.2.4 Alternative Measures of Per Capita Income

Table 20: Top 1% of Multiproduct Exporters

	Baseline	Nominal	PPP	GNI Atlas	GNI	Hous. Cons.
Log(Pc. Income)	0.113*** (0.037)	0.126*** (0.037)	0.207*** (0.058)	0.140*** (0.040)	0.149*** (0.044)	0.128*** (0.043)
Log(GDP)	0.161*** (0.023)	0.158*** (0.023)	0.152*** (0.022)	0.151*** (0.023)	0.154*** (0.025)	0.168*** (0.024)
Log(Distance)	-0.550*** (0.102)	-0.546*** (0.102)	-0.558*** (0.101)	-0.533*** (0.100)	-0.508*** (0.099)	-0.503*** (0.099)
Border	0.265 (0.168)	0.258 (0.166)	0.268 (0.167)	0.270 (0.165)	0.321* (0.162)	0.285* (0.155)
Comm. Language	0.549*** (0.124)	0.557*** (0.125)	0.580*** (0.129)	0.589*** (0.128)	0.664*** (0.130)	0.652*** (0.125)
Island	0.085 (0.097)	0.076 (0.098)	0.126 (0.091)	0.091 (0.097)	0.132 (0.099)	0.122 (0.103)
Landlocked	-0.167* (0.101)	-0.172* (0.100)	-0.132 (0.124)	-0.183* (0.099)	-0.132 (0.085)	-0.186* (0.102)
R^2	0.67	0.67	0.67	0.67	0.70	0.69
# Observations	4380	4380	4242	4326	4088	4141

Results from OLS of equation (48) for the Top 1% of Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Baseline uses real per capita GDP. Nominal uses nominal per capita GDP. PPP uses PPP adjusted real per capita GDP. GNI (Atlas) uses gross national income per capita (with the Atlas method). Hous. Cons. uses household final consumption.

Table 21: Top 5% of Multiproduct Exporters

	Baseline	Nominal	PPP	GNI Atlas	GNI	Hous. Cons.
Log(Pc. Income)	0.065*** (0.023)	0.073*** (0.023)	0.127*** (0.035)	0.080*** (0.024)	0.077*** (0.025)	0.068*** (0.025)
Log(GDP)	0.102*** (0.015)	0.101*** (0.015)	0.097*** (0.016)	0.097*** (0.016)	0.101*** (0.018)	0.107*** (0.017)
Log(Distance)	-0.357*** (0.072)	-0.356*** (0.073)	-0.366*** (0.075)	-0.352*** (0.072)	-0.346*** (0.073)	-0.342*** (0.071)
Border	0.357** (0.167)	0.353** (0.166)	0.351** (0.169)	0.354** (0.166)	0.371** (0.166)	0.356** (0.164)
Comm. Language	0.329*** (0.087)	0.335*** (0.088)	0.343*** (0.092)	0.346*** (0.091)	0.371*** (0.094)	0.364*** (0.092)
Island	0.046 (0.062)	0.042 (0.063)	0.061 (0.060)	0.047 (0.064)	0.060 (0.065)	0.057 (0.066)
Landlocked	-0.093 (0.057)	-0.098* (0.058)	-0.086 (0.077)	-0.101* (0.058)	-0.083 (0.056)	-0.098* (0.059)
R^2	0.59	0.59	0.59	0.59	0.61	0.60
# Observations	14157	14157	13738	14031	13564	13668

Results from OLS of equation (48) for the Top 5% of Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Baseline uses real per capita GDP. Nominal uses nominal per capita GDP. PPP uses PPP adjusted real per capita GDP. GNI (Atlas) uses gross national income per capita (with the Atlas method). Hous. Cons. uses household final consumption.

Table 22: Bottom 95% of Multiproduct Exporters

	Baseline	Nominal	PPP	GNI Atlas	GNI	Hous. Cons.
Log(Pc. Income)	0.027 (0.023)	0.034 (0.023)	0.055 (0.035)	0.038 (0.023)	0.029 (0.023)	0.034 (0.023)
Log(GDP)	0.056*** (0.014)	0.054*** (0.013)	0.055*** (0.014)	0.053*** (0.014)	0.057*** (0.016)	0.056*** (0.015)
Log(Distance)	-0.198*** (0.062)	-0.197*** (0.061)	-0.203*** (0.065)	-0.197*** (0.061)	-0.201*** (0.062)	-0.197*** (0.061)
Border	0.259* (0.149)	0.260* (0.148)	0.254* (0.152)	0.259* (0.149)	0.256* (0.151)	0.253* (0.152)
Comm. Language	0.150** (0.074)	0.155** (0.075)	0.158** (0.078)	0.158** (0.077)	0.157** (0.076)	0.159** (0.078)
Island	0.023 (0.043)	0.022 (0.043)	0.028 (0.042)	0.023 (0.044)	0.028 (0.045)	0.024 (0.046)
Landlocked	0.015 (0.033)	0.013 (0.033)	0.038 (0.034)	0.014 (0.033)	0.019 (0.035)	0.015 (0.034)
R^2	0.60	0.60	0.60	0.60	0.60	0.60
# Observations	80718	80718	79552	80428	79874	80021

Results from OLS of equation (48) for the Bottom 95% of Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Baseline uses real per capita GDP. Nominal uses nominal per capita GDP. PPP uses PPP adjusted real per capita GDP. GNI (Atlas) uses gross national income per capita (with the Atlas method). Hous. Cons. uses household final consumption.

2.2.5 Alternative distributions of firms

Figure 5: Product Scope and Sales of Mexican MPF (2004)

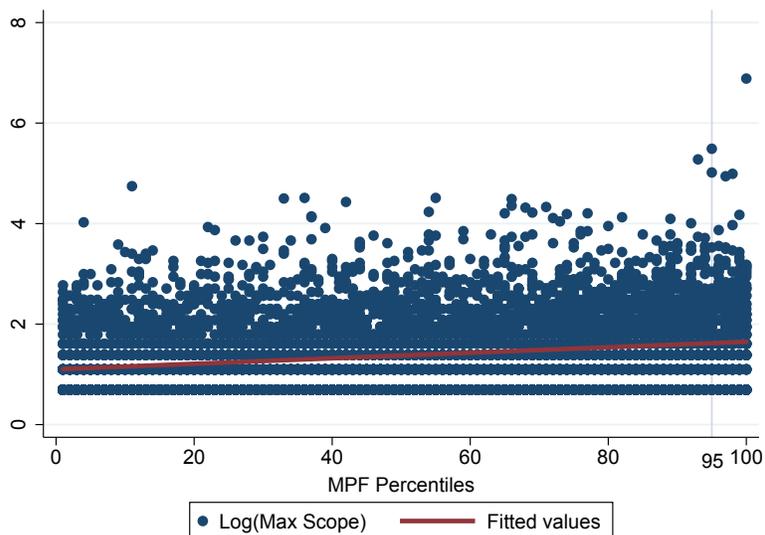


Figure 6: Destinations and Sales of Mexican MPF (2004)

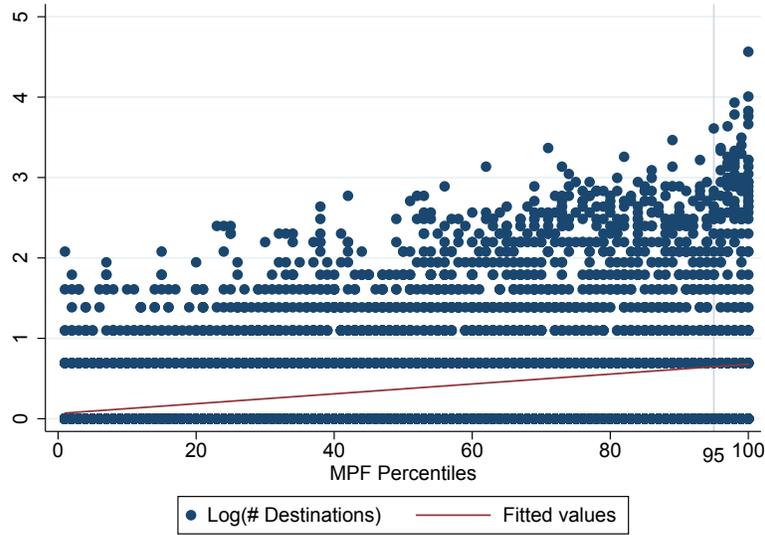


Table 23: Per Capita Income and Product Scope of Mexican Exporters

	Lagged Percentiles			Percentiles in US		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
Log(Pc.income)	0.028 (0.023)	0.066*** (0.024)	0.126*** (0.041)	0.018 (0.023)	0.109*** (0.031)	0.164*** (0.052)
logGDP	0.056*** (0.014)	0.116*** (0.016)	0.174*** (0.024)	0.059*** (0.015)	0.138*** (0.018)	0.208*** (0.030)
Log(Distance)	-0.215*** (0.063)	-0.381*** (0.074)	-0.574*** (0.111)	-0.185*** (0.064)	-0.427*** (0.088)	-0.592*** (0.119)
Border	0.270* (0.153)	0.373** (0.164)	0.204 (0.168)	0.300* (0.155)	0.387** (0.166)	0.195 (0.177)
Comm. Language	0.155** (0.074)	0.362*** (0.091)	0.579*** (0.134)	0.170** (0.078)	0.418*** (0.106)	0.710*** (0.158)
Island	0.030 (0.043)	0.027 (0.066)	0.102 (0.101)	0.016 (0.045)	0.066 (0.077)	0.135 (0.120)
Landlocked	0.016 (0.038)	-0.116* (0.066)	-0.193* (0.109)	-0.008 (0.036)	-0.087 (0.093)	-0.176 (0.156)
R^2	0.60	0.59	0.67	0.61	0.64	0.71
# Observations	61939	9502	3153	59502	7965	2656

Results from OLS of equation (48). Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. In the first three column exporters are divided into bottom 95%, top 5% and top 1% by their sales in the previous year. In the second three columns firms are divided in groups according to their annual sales in the US. In the last three columns I divide firms by percentiles within each industry.

Table 24: Per Capita Income and Product Scope of Mexican Exporters

	Bottom 95%	Top 5%	Top 1%
Log(Pc.income)	0.016 (0.015)	0.042** (0.019)	0.082*** (0.025)
logGDP	0.036*** (0.009)	0.083*** (0.013)	0.126*** (0.017)
Log(Distance)	-0.130*** (0.042)	-0.286*** (0.061)	-0.392*** (0.069)
Border	0.136 (0.090)	0.333** (0.145)	0.285** (0.136)
Comm. Language	0.100** (0.049)	0.241*** (0.076)	0.455*** (0.086)
Island	0.023 (0.027)	0.034 (0.048)	0.044 (0.069)
Landlocked	0.005 (0.021)	-0.085* (0.043)	-0.126* (0.072)
R^2	0.62	0.61	0.63
# Observations	135953	24031	7908

Results from OLS of equation (48). Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. The first three columns consider all Mexican firms (not only the Multiproduct ones). In the last three columns, new entrants and firm that exit are dropped prior to the division in percentiles.

2.3 Which Products Are Sold in Poor and Rich Economies?

While the previous section showed that firms sell more varieties in richer economies, this section considers which particular varieties are sold. A corollary of the previous testable prediction is that firms tend to sell their core varieties to all locations while they export their non-core varieties only to richer economies.

Let us start by considering the Exporter Dynamics Database. For each firm, I select the most successful variety by total export value. I define a variety “Core” if its total sales are more than a quarter of the sales of the most successful variety. Otherwise the variety is Non-Core¹². I count the number of Core and Non-Core varieties exported per firm in each destination, and run regression (48) on the two types of varieties separately. Table 25 illustrates the results.

While more Core varieties are sold in richer economies, Non-Core varieties are far more sensitive to changes in the per capita income of the destination. Doubling the per capita income of the destination increases the number of Core varieties by 2.4%, and the number of Non-Core varieties by 17% for the top 1% of Mexican Multiproduct Exporters.

Since we do not have information on sales of each product for the scraped data, dividing varieties in Core and Non-Core is quite challenging. Ranking varieties according to the number of destinations reached generates similar results. However, such a finding could seem obvious

¹²Results are robust to changes in the threshold.

Table 25: Core vs. Non-Core Varieties and Per Capita Income

	Core	Non-Core
Log(Pc.income)	0.024*** (0.007)	0.176*** (0.061)
logGDP	0.024*** (0.005)	0.246*** (0.033)
Log(Distance)	-0.109*** (0.014)	-0.809*** (0.154)
Border	0.047* (0.025)	-0.107 (0.234)
Comm. Language	0.075*** (0.024)	0.806*** (0.182)
Island	-0.006 (0.017)	0.155 (0.134)
Landlocked	-0.028 (0.044)	-0.135 (0.166)
R^2	0.52	0.70
# Observations	3693	2505

Results from OLS of equation (48) by type of variety. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Dependent variable: log of number of Core (Non-Core) consumption varieties per firm per destination. Sample: top 1% of Mexican Multiproduct Exporters.

given that we established in the previous section that these firms sell more varieties in richer economies. In addition, a well-known fact on multiproduct firms is that their core goods are sold in all destinations ([Arkolakis et al., 2014](#)).

However, we can consider the Samsung database for which I have five distinct categories of goods: a group of Core goods (Smartphones, Other phones) and Non-Core Accessories. Given that Tablets and Wearables are relatively new products it is probably wise not to label them as Core and Non-Core. I count the number of varieties in each category separately and run regression (48). Table 26 shows that Samsung offers Core varieties independently of the level of development of a country. Only for Tablets, the relationship is significant at the 90% level. However, the number of accessories is highly sensitive to the per capita income of the destination and it drives the result that more varieties are offered in richer economies.

Table 26: Per Capita Income and Samsung’s Core and Non-Core Varieties

	Accessories	Smartphones	Other phones	Tablets	Wearables
Log(Pc.Income)	0.345*** (0.126)	-0.072 (0.090)	0.015 (0.132)	0.107* (0.062)	-0.018 (0.053)
Log(GDP)	0.137 (0.097)	-0.017 (0.069)	-0.140 (0.108)	-0.154*** (0.048)	0.088** (0.041)
Island	-0.421 (0.426)	-0.116 (0.303)	0.186 (0.445)	-0.118 (0.210)	-0.009 (0.181)
Landlocked	0.173 (0.435)	-0.343 (0.310)	-0.357 (0.473)	-0.347 (0.214)	0.173 (0.185)
Tariff	-0.058 (0.046)	0.014 (0.033)	0.029 (0.058)	0.023 (0.023)	-0.053*** (0.019)
R^2	0.29	0.05	0.05	0.22	0.21
# Observations	49	50	47	50	50

Results from OLS of equation (48). ***: significant at 99%, ** at 95%, * at 90%. Dependent variable: log of number of products offered online per category per destination. Results are reported by category. The category "Other Phones" is missing for Brazil, Colombia, and United States. The category "Accessories" is missing for Iran.

2.4 Robustness Checks for the Second Stylized Fact

The second stylized fact that I document using Mexican data is the non-monotone hump shaped relationship between the scope of an exporter and its market share. The baseline empirical model is given by:

$$\ln(\# \text{ Products}_{kMjt}) = \beta_s \ln(1 + s_{kMjt}) + \beta_{s^2} (\ln(1 + s_{kMjt}))^2 + f_k + d_{jt} + y_t + \epsilon_{kMjt} \quad (49)$$

where s_{kMjt} is the market share of the firm, defined as the share of firm k sales to a destination j over the total Mexican export to country j in the industry of firm k ¹³. f_k is a firm fixed effect, d_{jt} is a destination-year fixed effect, and y_t is a year fixed effect. In the paper I use the a two stage approach to avoid the multicollinearity between the $\ln(1 + s_{kMjt})$ and $(\ln(1 + s_{kMjt}))^2$ (Montgomery et al., 2013). In particular I estimate the following regression:

$$(\ln(1 + s_{kMjt}))^2 = \alpha_s \ln(1 + s_{kMjt}) + f_k + y_t + \eta_{kMjt} \quad (50)$$

and use the estimated error $\hat{\eta}_{kMjt}$ in (49):

$$\ln(\# \text{ Products}_{kMjt}) = \beta_s \ln(1 + s_{kMjt}) + \beta_{s^2} \hat{\eta}_{kMjt} + f_k + d_j + \epsilon_{kMjt} \quad (51)$$

For the three group of multiproduct firms that I consider (bottom 95%, top 5%, and top 1%) I normalize the market share dividing it by the average in the group, in order to have comparable

¹³I normalize s_{kMjt} dividing it by the average s_{kMjt} in the bin considered

coefficients across groups. I find a positive β_s and a negative β_{s^2} for each of the three group. Moreover I use the [Lind and Mehlum \(2010\)](#) test for non monotone hump shaped relationship.

The [Lind and Mehlum \(2010\)](#) test works as follows. The null hypothesis is that the relationship is monotone or U-shaped, and the alternative is that it is hump-shaped. The null hypothesis is rejected if either or both the following conditions are rejected:

$$\begin{aligned}\beta_s + 2\beta_{s^2} \ln(1 + s_L) &\leq 0 \\ \beta_s + 2\beta_{s^2} \ln(1 + s_H) &\geq 0\end{aligned}$$

where s_L and s_H are some lower and upper bounds. We reject the null hypothesis if the slope of the curve is negative at the beginning and/or positive at the end. I choose for the lower bound s_L , the minimum value of market share in the sample, the 5th percentile, and the 10th percentile. For the upper bound s_H I choose the maximum, the 95th percentile and the 90th percentile. The hump-shaped relationship is confirmed (Table 27) and the results are especially robust for the top 5% and 1% of Mexican multiproduct exporters.

Table 27: Multiproduct Firms and their Market Share

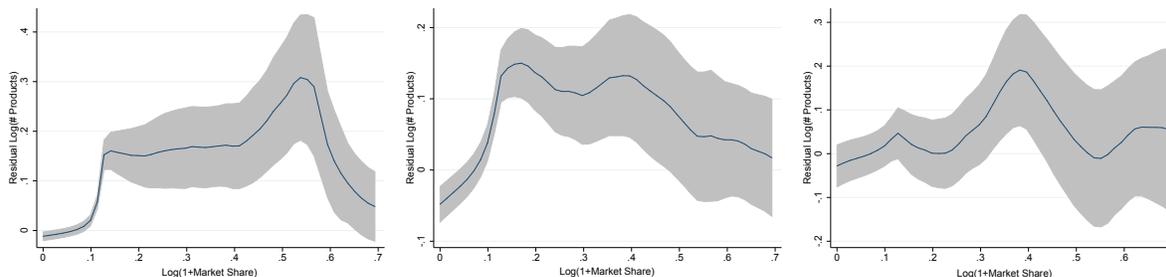
	Bottom 95%	Top 5%	Top 1%
s_{kij}	0.209*** (0.012)	0.343*** (0.025)	0.318*** (0.056)
s_{kij}^2	-0.148*** (0.014)	-0.478*** (0.047)	-0.497*** (0.099)
R^2	0.63	0.69	0.82
# Observations	82602	14184	4224
Hump-Shaped t-value {min, max}	8.87***	8.50***	3.84***
Hump-Shaped t-value {5 th pct, 95 th pct}	5.97***	8.16***	3.73***
Hump-Shaped t-value {10 th pct, 90 th pct}	0.98	7.47***	3.48***

Results from OLS of equation (49). Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. s_{kMjt} and s_{kMjt}^2 are normalized by their year sample average. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\text{min, max}\}$, $\{5^{th}\text{pct, }95^{th}\text{pct}\}$ and $\{10^{th}\text{pct, }90^{th}\text{pct}\}$. N.A.: the extremum is outside the sample. In the Hump-Shaped test for $\{5^{th}\text{pct, }95^{th}\text{pct}\}$ and $\{10^{th}\text{pct, }90^{th}\text{pct}\}$ I drop the destinations served by only one firm.

An additional test of the non-monotone, hump shaped relationship is to use local polynomial regressions. For each group of firms, I regress $\ln(\# \text{ Products}_{kMjt})$ on firm and destination-year fixed effects and record the residual. Then I plot the local polynomial relationship between such residual and $\ln(1 + s_{kMjt})$ for the year 2005. Figure 7 shows the result. For each group of firms, there is a non-monotone hump shaped relationship. The presence of large market shares for the bottom 95% of exporters should not surprise, as few of them tend to be the only exporters in a given industry and destination. Following [Robinson \(1988\)](#), I repeat the analysis, by plotting, for the year 2005, the local polynomial relationship between the residuals from regressing $\ln(\# \text{ Products}_{kMjt})$ on firm and destination-year fixed effects, on the residuals from regressing $\ln(1 + s_{kMjt})$ on firm and destination-year fixed effects. Although the hump shaped relationship is less prominent, the

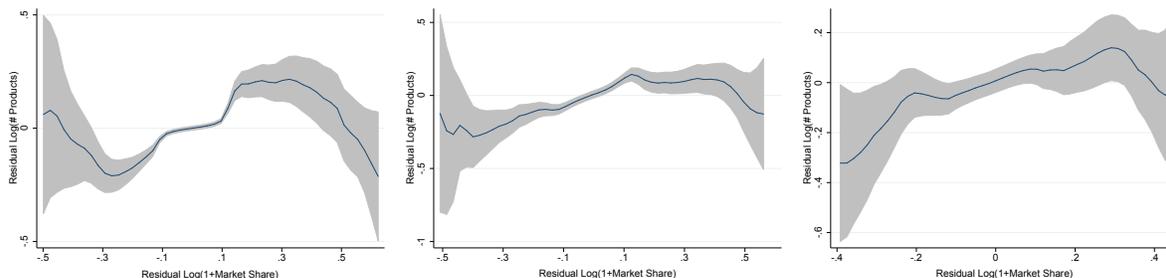
results are robust to this alternative specification (Figure 8).

Figure 7: Residual Scope on Market Share: Local Polynomial Smoothing



In order: Bottom 95%, Top 5%, and Top 1%. Alternative Epanechnikov kernel function, with bandwidth=0.12 and degree=0. The gray area is the 95% C.I.

Figure 8: Residual Scope on Residual Market Share: Local Polynomial Smoothing



In order: Bottom 95%, Top 5%, and Top 1%. Alternative Epanechnikov kernel function, with bandwidth=0.12 and degree=0. The gray area is the 95% C.I.

This section illustrates several robustness checks that confirm the stylized fact. Section 2.4.1 shows that including firm-year fixed effects does not alter the results (Table 28). Results still hold if we focus on 2004, although $\beta_{s,2}$ is not significant for the top 1% of exporters, possibly because of the small number of observations. Table (29) shows the result if we ignore the first stage and estimate (49). Because of multicollinearity, the coefficient β_s almost doubles relative to the baseline case. The last three columns of (29) shows the results of the regressions if we use s_{kMjt} and s_{kMjt}^2 as dependent variables.

Section 2.4.2 reports the result from estimating (49) for all countries in the Exporter Dynamics Database (using origin-destination fixed effects), and for Peru. The results are remarkably similar to those obtained with Mexican data.

The third set of robustness checks involves different definitions of market share. The market share is defined as the sales of firm k in a destination j , over the following alternative measures:

1. The industry specific export of Mexico in j , where an industry is an 2 digit SITC Rev 2 (First three columns of table 31)

2. The industry specific imports of j , where an industry is an 2 digit SITC Rev 2 (Last three columns of table 31). I use data from [Feenstra et al. \(2005\)](#) on the industry level imports at SITC 2 digit level.
3. The total imports of j (First three columns of table 32)
4. The total household consumption in j (Last three columns of table 32)

Section 2.4.4 shows that results are robust to different distributions of firms. Table 33 divides firms in percentiles within industries, using the previous year's sales and using the sales in the US, and finds similar results.

2.4.1 Robustness Checks

Table 28: Product Scope of Mexican multiproduct firms and market share

	Firm-Year FE			2004		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
s_{kMjt}	0.203*** (0.013)	0.349*** (0.026)	0.305*** (0.058)	0.174*** (0.019)	0.295*** (0.041)	0.192** (0.084)
s_{kMjt}^2	-0.147*** (0.012)	-0.483*** (0.047)	-0.522*** (0.099)	-0.157*** (0.016)	-0.494*** (0.066)	-0.345** (0.144)
R^2	0.58	0.70	0.83	0.57	0.68	0.82
# Observations	56666	12980	4028	8657	1920	594
Hump-Shaped t-value	10.18***	8.48***	4.06***	8.51***	6.08***	1.92**
Hump-Shaped t-value 5-95p	6.91***	8.13***	3.96***	6.32***	5.84***	1.88**
Hump-Shaped t-value 10-90p	1.35*	7.41***	3.71***	2.20**	5.35***	1.77**

Results from OLS of equation (49) for the bottom 95%, top 5% and 1% of Mexican Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\min, \max\}$, $\{5^{th} \text{pct}, 95^{th} \text{pct}\}$ and $\{10^{th} \text{pct}, 90^{th} \text{pct}\}$. N.A.: the extremum is outside the sample.

Table 29: Product Scope of Mexican multiproduct firms and market share

	No First Stage			No Log		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
s_{kMjt}	0.622*** (0.046)	1.217*** (0.093)	1.095*** (0.148)	0.014*** (0.001)	0.081*** (0.009)	0.100*** (0.025)
s_{kMjt}^2	-0.148*** (0.014)	-0.478*** (0.047)	-0.497*** (0.099)	-0.002*** (0.000)	-0.052*** (0.005)	-0.090*** (0.016)
R^2	0.63	0.69	0.82	0.62	0.68	0.82
# Observations	82602	14184	4224	82602	14184	4224
Hump-Shaped t-value	7.56***	6.92***	2.84***	10.92***	9.33***	3.94***
Hump-Shaped t-value 3-97p	N.A.	5.52***	2.47***	N.A.	8.69***	3.91***
Hump-Shaped t-value 5-95p	N.A.	0.59	1.38*	N.A.	7.19***	3.82***

Results from OLS of equation (49) for the bottom 95%, top 5% and 1% of Mexican Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\min, \max\}$, $\{5^{th} \text{pct}, 95^{th} \text{pct}\}$ and $\{10^{th} \text{pct}, 90^{th} \text{pct}\}$. N.A.: the extremum is outside the sample.

2.4.2 Alternative Countries of Origin

Table 30: Product Scope of multiproduct firms and market share

	All countries			Peru		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
s_{kijt}	0.128*** (0.008)	0.226*** (0.013)	0.222*** (0.029)	0.235*** (0.013)	0.418*** (0.034)	0.439*** (0.036)
s_{kijt}^2	-0.236*** (0.012)	-0.612*** (0.038)	-0.951*** (0.058)	-0.274*** (0.022)	-0.556*** (0.044)	-0.797*** (0.068)
R^2	0.59	0.57	0.66	0.62	0.54	0.62
# Observations	249713	48145	12579	59898	15423	3922
Hump-Shaped t-value	15.66***	14.76***	7.69***	10.88***	11.13***	9.75***
Hump-Shaped t-value 5-95p	15.66***	14.51***	7.55***	9.79***	10.73***	9.26***
Hump-Shaped t-value 10-90p	15.65***	14.06***	7.11***	7.90***	10.11***	8.68***

Results from OLS of equation (49) for the bottom 95%, top 5% and 1% of Mexican Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\min, \max\}$, $\{5^{th}\text{pct}, 95^{th}\text{pct}\}$ and $\{10^{th}\text{pct}, 90^{th}\text{pct}\}$. N.A.: the extremum is outside the sample.

2.4.3 Alternative Definitions of Market Share

Table 31: Scope and market share

	Sales over SITC 2 dig. Export			Sales over SITC 2 dig. Import		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
s_{kMjt}	0.185*** (0.010)	0.334*** (0.023)	0.324*** (0.051)	0.361*** (0.017)	0.533*** (0.026)	0.520*** (0.044)
s_{kMjt}^2	-0.161*** (0.013)	-0.571*** (0.061)	-0.509*** (0.119)	-0.093*** (0.015)	-0.143*** (0.021)	-0.133*** (0.023)
R^2	0.63	0.69	0.82	0.64	0.71	0.83
# Observations	82602	14184	4224	82367	14106	4191
Hump-Shaped t-value	10.28***	7.93***	3.25***	4.80***	4.40***	3.46***
Hump-Shaped t-value 5-95p	8.38***	7.82***	3.22***	N.A.	N.A.	N.A.
Hump-Shaped t-value 10-90p	3.91***	7.52***	3.13***	N.A.	N.A.	N.A.

Results from OLS of equation (49) for the bottom 95%, top 5% and 1% of Mexican Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\min, \max\}$, $\{5^{th}\text{pct}, 95^{th}\text{pct}\}$ and $\{10^{th}\text{pct}, 90^{th}\text{pct}\}$. N.A.: the extremum is outside the sample. The columns describe the definition of the market share used in the regressions.

Table 32: Scope and market share

	Sales over Total Imports			Sales over Household Consumption		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
s_{kMjt}	0.416*** (0.024)	0.548*** (0.024)	0.461*** (0.043)	0.404*** (0.020)	0.545*** (0.024)	0.466*** (0.043)
s_{kMjt}^2	-0.127*** (0.024)	-0.207*** (0.023)	-0.205*** (0.034)	-0.110*** (0.022)	-0.198*** (0.028)	-0.187*** (0.038)
R^2	0.65	0.71	0.83	0.65	0.71	0.83
# Observations	82602	14184	4224	82549	14140	4204
Hump-Shaped t-value	4.31***	6.88***	4.71***	4.00***	5.27***	3.62***
Hump-Shaped t-value 5-95p	N.A.	2.02**	2.17**	N.A.	1.37*	1.39*
Hump-Shaped t-value 10-90p	N.A.	N.A.	0.57	N.A.	N.A.	N.A.

Results from OLS of equation (49) for the bottom 95%, top 5% and 1% of Mexican Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\min, \max\}$, $\{5^{th} \text{pct}, 95^{th} \text{pct}\}$ and $\{10^{th} \text{pct}, 90^{th} \text{pct}\}$. N.A.: the extremum is outside the sample. The columns describe the definition of the market share used in the regressions.

2.4.4 Alternative Distributions for Firms

Table 33: Scope and market share

	Lagged Percentiles			US Percentiles			Perc. by Industry		
	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%	Bottom 95%	Top 5%	Top 1%
s_{kMjt}	0.208*** (0.013)	0.352*** (0.029)	0.361*** (0.063)	0.208*** (0.013)	0.352*** (0.029)	0.361*** (0.063)	0.151*** (0.011)	0.293*** (0.021)	0.373*** (0.040)
s_{kMjt}^2	-0.158*** (0.015)	-0.481*** (0.054)	-0.376*** (0.117)	-0.158*** (0.015)	-0.481*** (0.054)	-0.376*** (0.117)	-0.094*** (0.011)	-0.343*** (0.034)	-0.651*** (0.078)
R^2	0.63	0.70	0.83	0.63	0.70	0.83	0.64	0.67	0.76
# Observations	63441	9585	3030	63441	9585	3030	138255	24512	6988
HS	8.90***	7.33***	2.12***	8.90***	7.33***	2.12**	7.35***	8.52**	6.89***
HS 5-95p	6.62***	7.07***	2.06***	6.62***	7.07***	2.06**	3.87***	8.06***	6.73***
HS 10-90p	2.54***	6.50***	1.87**	2.54***	6.50***	1.87**	N.A.	7.21***	6.41***

Results from OLS of equation (49) for the bottom 95%, top 5% and 1% of Mexican Multiproduct Exporters. Robust std. error in parenthesis. Cluster: destination country. ***: significant at 99%, ** at 95%, * at 90%. Hump-shape t-test: t-value and significance of [Lind and Mehlum \(2010\)](#) test evaluated at $s_{kMjt} = \{\min, \max\}$, $\{5^{th} \text{pct}, 95^{th} \text{pct}\}$ and $\{10^{th} \text{pct}, 90^{th} \text{pct}\}$. N.A.: the extremum is outside the sample. The columns describe the assumption on the distribution of firms used in the regressions.

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